

Mensuration - Area and perimeter of square, rectangle and parallelogram

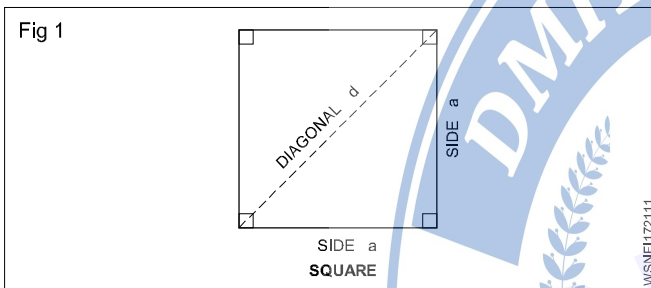
In Engineering field, an Engineer has to estimate the material, manpower, machinery, etc. required to prepare the geometrical objects. Hence we must be very conversant with all relevant formulae connected with geometrical objects.

- Length - l unit
- Breadth or width - b unit
- Diagonal - d unit
- Diameter - d unit

- Radius - r unit
- Semi perimeter - S unit
- Perimeter - P unit
- Circumference - C unit
- Area - A unit<sup>2</sup>
- Total surface area - T.S.A unit<sup>2</sup>
- Lateral surface area - L.S.A unit<sup>2</sup>
- Volume - V unit<sup>3</sup>

Square

This is also a four sided figure, opposite sides are parallel. All the four sides are equal. Angle between adjacent side are 90°.



Area (A) = a<sup>2</sup>  
 = 18 x 18 = 324 cm<sup>2</sup>  
 Perimeter of square = 72 cm

Diagonal = 25.45 cm ; Area = 324 cm<sup>2</sup>

2 If the diagonal of a square measure 10 cm. Find area of the square.

Diagonal of the square (d) = √2 a = 10 cm

Side (a) = d / √2

Area (a<sup>2</sup>) = d / √2 x d / √2 = d<sup>2</sup> / 2

= 10<sup>2</sup> / 2 = 100 / 2

= 50 cm<sup>2</sup>

= 50 cm<sup>2</sup>

Area of the square

3 The perimeter of one square is 748 cm and that of Another is 336 cm. Find the perimeter of a square and total area of the two squares.

Side of the square (a) = Perimeter / 4

1<sup>st</sup> square

Side (a) = Perimeter of 1<sup>st</sup> square / 4

= 748 / 4 = 187 cm

Area (A)

= a<sup>2</sup>  
 = 187 x 187  
 = 34,969 cm<sup>2</sup>

2<sup>nd</sup> square

Side (a) = Perimeter of 2<sup>nd</sup> square / 4

= 336 / 4 = 84 cm

A = a<sup>2</sup> (or) unit<sup>2</sup>

P = 4a unit

d = √2 a unit

a = d / √2 unit where √2 = 1.414

Find the area of a brass sheet in the form of a square whose perimeter is 31.2 cm.

Perimeter(P) = 4a = 31.2 cm

∴ a = 31.2 / 4 = 7.8 cm

Area (A) = a<sup>2</sup>  
 = 7.8 x 7.8 = 60.84 cm<sup>2</sup>

Examples

1 Find out the circumference, diagonal and area of a square, whose side is 18 cm.

Side of the square (a) = 18 cm

Perimeter (P) = 4a  
 = 4 x 18 = 72 cm

Diagonal (d) = √2 x a  
 = √2 x 18 = 1.414 x 18  
 = 25.45 cm

$$\begin{aligned} \text{Area (A)} &= a^2 \\ &= 84 \times 84 \\ &= 7,056 \text{ cm}^2 \end{aligned}$$

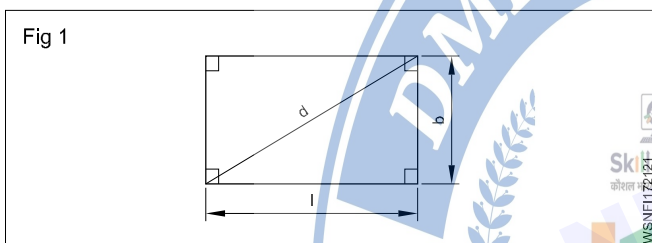
$$\begin{aligned} \text{Total area of two squares} &= 34,969 + 7,056 \\ &= 42,025 \text{ cm}^2 \end{aligned}$$

## Assignment

- Find the Area, Perimeter and diagonal of a square steel plate whose side measures 28.1 cm.
- Find the area of a square whose diagonal is equal to 8.5 cm.
- Find the area of the square if the side of the square is 28 cm.
- Find its side if the area of the square field is 169 m<sup>2</sup>.
- Find the area of the square if the diagonal of the square is 20 cm.
- Find the perimeter of a square whose diagonal is 144 m.
- Find the area if the perimeter of a square plot is 48 m.

## Rectangle

This is a four sided figure. Opposite sides are parallel. Angles between adjacent sides are 90°.



$$A = \text{Area} = \text{length} \times \text{breadth} = l \cdot b \cdot \text{unit}^2$$

$$P = \text{Perimeter} = 2(l + b) \text{ unit}$$

$$\text{Diagonal} = \sqrt{l^2 + b^2} \text{ unit}$$

### Examples

- Find the Area, Perimeter and diagonal of a rectangle whose length and breadth are 144 mm and 60 mm respectively.

$$\begin{aligned} \text{Area} = A &= l \times b \text{ unit}^2 \\ &= 144 \times 60 = 8640 \text{ mm}^2 \end{aligned}$$

$$\begin{aligned} \text{Perimeter} = P &= 2(l + b) \text{ unit} \\ &= 2(144 + 60) \\ &= 2 \times 204 = 408 \text{ mm} \end{aligned}$$

$$\begin{aligned} \text{Diagonal} = d &= \sqrt{l^2 + b^2} \text{ unit} \\ &= \sqrt{144^2 + 60^2} \\ &= \sqrt{20736 + 3600} \\ &= \sqrt{24336} = 156 \text{ mm} \end{aligned}$$

- The perimeter of a rectangle is equal to 42 cm. If its breadth is 9 cm. Find the length of the rectangle.

$$\text{Perimeter} = 42 \text{ cm}$$

$$\text{Breadth} = 9 \text{ cm}$$

$$\text{Length} = ?$$

$$\text{Perimeter} = P = 2(l + b)$$

$$2(l + 9) = 42$$

$$l + 9 = 42 \div 2$$

$$l + 9 = 21$$

$$l = 21 - 9$$

$$l = 12 \text{ cm}$$

- The perimeter of a rectangle is 48 cm and its length is 4 cm more than its width. Find the length and breadth of the rectangle.

$$\text{Perimeter (P)} = 48 \text{ cm}$$

$$\text{Breadth (b)} = x$$

$$\text{Length (l)} = x + 4$$

$$2(l + b) = \text{Perimeter}$$

$$2(x + 4 + x) = 48$$

$$2(2x + 4) = 48$$

$$4x + 8 = 48$$

$$4x = 48 - 8$$

$$x = \frac{40}{4} = 10$$

$$x = \text{breadth} = 10 \text{ cm}$$

$$\text{length} = x + 4 = 10 + 4 = 14 \text{ cm}$$

- 4 How many rectangular pieces of 50 cm x 20 cm can be cut out from a sheet of 1000 cm x 500 cm.

Sheet size = 1000 cm x 500 cm

Size of the rectangular piece to be cut = 50 cm x 20 cm

$$\text{No. of pieces to be cut in lengthwise} = \frac{1000}{50} = 20$$

$$\text{No. of pieces to be cut in breadthwise} = \frac{500}{20} = 25$$

$$\begin{aligned} \text{Total no. of pieces to be cut out} &= 20 \times 25 \\ &= 500 \end{aligned}$$

- 5 The perimeter of a rectangle is 320 metre. Its sides are in the ratio of 5:3. Find the area of the rectangle.

$$\text{Ratio} = 5:3 = l : b$$

$$\text{length } l = 5x$$

$$\text{breadth } b = 3x$$

$$2(l + b) = \text{Perimeter}$$

$$2(5x + 3x) = 320$$

$$2(8x) = 320$$

$$16x = 320$$

$$x = \frac{320}{16} = 20$$

$$l = 5x = 5 \times 20 = 100 \text{ m}$$

$$b = 3x = 3 \times 20 = 60 \text{ m}$$

$$\begin{aligned} \text{Area} &= l \times b \quad (\text{length} = 100\text{m, breath} = 60\text{m}) \\ &= 100 \times 60 \end{aligned}$$

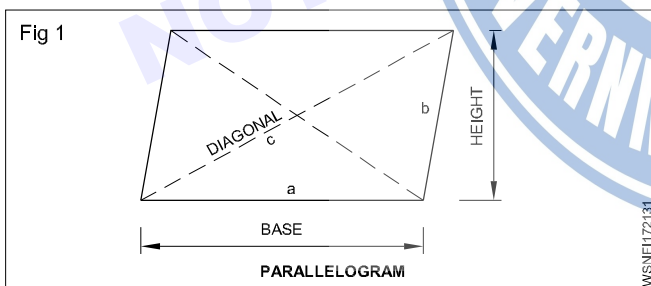
$$\text{Area} = 6000 \text{ m}^2$$

## Assignment

- Find the area of a rectangular plot whose sides are 24 metres and 20 metres respectively. Also find the perimeter of the plot.
- How many rectangular pieces of 5 cm x 4 cm will you get out of 65 cm x 30 cm brass sheet?
- Find its breadth and area if the perimeter of a rectangle is 400 metre and its length is 140 m.
- Find its area, if the opposite sides of a rectangle are 64 cm and 25 cm respectively.
- What is the width of the rectangle if a rectangle has an area of 224 cm<sup>2</sup> and length 16 cm.
- What is the length of the diagonal of a rectangle with sides 16 cm and 12 cm?
- Find the area of the rectangle if the perimeter of the rectangle is 100 cm and the ratio of its length and breadth is 3:2.

## Parallelogram

This is also a four sided figure, opposite side being parallel to each other.



Area of parallelogram = base x height

$$\text{or } = 2x\sqrt{s(s-a)(s-b)(s-c)}$$

Where

$$s = \frac{a+b+c}{2}$$

a and b are adjacent sides.

$$P = 2(a+b)$$

### Examples

- 1 The base and height of a parallelogram are 7.1 cm and 2.85 cm. Calculate its area.

$$\begin{aligned} A &= \text{base} \times \text{height units}^2 \\ &= 7.1 \times 2.85 \\ &= 20.235 \text{ cm}^2 \end{aligned}$$

- 2 Find the height of a parallelogram whose area is 20 cm<sup>2</sup> and base is 10 cm.

$$\begin{aligned} A &= \text{base} \times \text{height units}^2 \\ h &= \frac{\text{area}}{\text{base}} \\ &= \frac{20}{10} \\ &= 2 \text{ cm} \end{aligned}$$

3. Two sides of a parallelogram are 12 cm and 8 cm. The diagonal is 10 cm long. Find the area of the parallelogram.

$$\text{Area } A = 2x\sqrt{s(s-a)(s-b)(s-c)} \text{ units}^2$$

$$s = \frac{a+b+c}{2}$$

$$= \frac{12+8+10}{2}$$

$$= \frac{30}{2}$$

$$= 15$$

$$A = 2x\sqrt{15(15-12)(15-8)(15-10)}$$

$$= 2x\sqrt{15 \times 3 \times 7 \times 5}$$

$$= 2x\sqrt{1575}$$

$$= 2 \times 39.686$$

$$\text{Area } A = 79.37 \text{ cm}^2$$

## Assignment

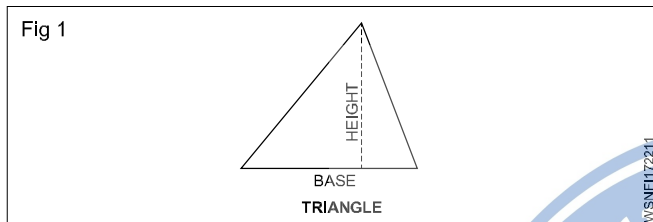
- Find the area of a parallelogram, if its base and height are 8.1 cm and 30.8 cm respectively.
- Find the area of a parallelogram, if the sides of a field in the shape of parallelogram are 12 m and 17 m and one of the diagonal is 25 m.
- Find the base of a parallelogram whose height is 12 cm and area is 120 cm<sup>2</sup>.
- Find the height of a parallelogram whose base is 40 cm and area is 320 cm<sup>2</sup>.
- Find the area of the land if the sides of a land in the shape of a parallelogram are 24 m and 28 m respectively and one of the diagonal is 30 m.
- What is the perimeter of parallelogram if base is 10 cm and other side is 5 cm?
- Find the area of parallelogram if its base and height are 25 cm and 12 cm.
- Find the base of a parallelogram if height is 15 cm and area is 150 cm<sup>2</sup>.
- Find the area of parallelogram if side is 5 cm, diagonal is 8 cm and diagonal bisects each other at right angles.
- Find the height of a parallelogram if base is 80 cm and area is 640 cm<sup>2</sup>.
- Find the area of parallelogram if its base and height are 15 cm and 8 cm.
- Calculate the perimeter and area of parallelogram if base, height are 12.7 cm, 5.5 cm and other side is 6.5 cm
- Find the height of parallelogram if the area is 20 cm<sup>2</sup> and base is 10 cm

Mensuration - Area and perimeter of triangles

Triangles

Tri means three. Hence tri- angle means three angled figure. For construction of three angled figure, there should be three sides. Hence triangle means three sided figure. Sum of the three angles of any triangle = 180°.

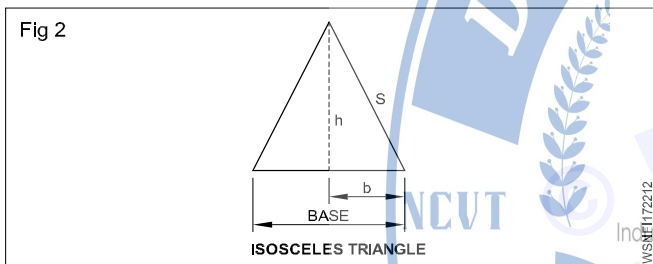
i Any triangle.



Area of any triangle =  $\frac{1}{2}$  x Base x Height unit<sup>2</sup>

ii Isosceles Triangle

In this triangle two of its sides are equal.



Area of isosceles triangle =  $\frac{1}{2}$  x Base x Height

Where

base = 2.b

s = One of equal sides (or) Slant height

h = Height =  $\sqrt{s^2 - b^2}$

Area of isosceles triangle =  $\frac{1}{2}$  x 2b x  $\sqrt{s^2 - b^2}$

= b .  $\sqrt{s^2 - b^2}$  unit<sup>2</sup>

(Where b= half of base)

(or) Area of Isosceles triangle =  $\frac{1}{4}$  b  $\sqrt{4a^2 - b^2}$  unit<sup>2</sup>

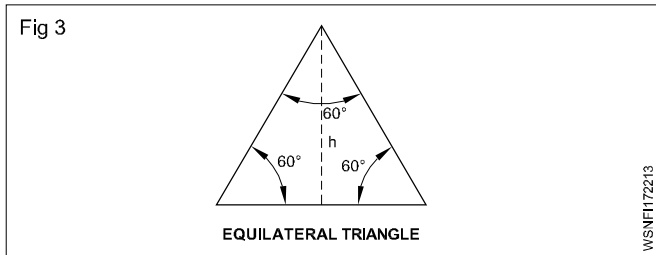
a = Equal sides

b = Base

iii Equilateral triangle

In this triangle all the three sides are equal. Hence angle between adjacent sides is 60° (because no. of angles)

total = 180° ; angle between sides =  $\frac{180}{3} = 60^\circ$



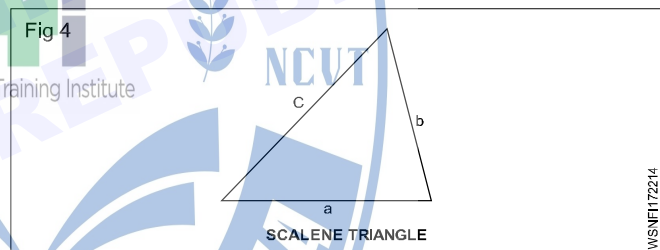
Area of equilateral triangle =  $\frac{\sqrt{3}}{4}$  x side<sup>2</sup>  
 =  $\frac{\sqrt{3}}{4}$  x a<sup>2</sup> unit<sup>2</sup>

Where  $\sqrt{3} = 1.732$   
 Perimeter P = 3a unit

P =  $\frac{\sqrt{3}}{2}$  a unit

iv Scalene triangle

In this triangle the sides are not equal. Angles between the sides, are also not equal. we may also call this triangle as irregular triangle.



Area of triangle =  $\sqrt{s(s-a)(s-b)(s-c)}$  unit<sup>2</sup>

where

a,b,c are sides of triangle

s = Semi perimeter =  $\frac{a+b+c}{2}$  unit

v Right angled triangle

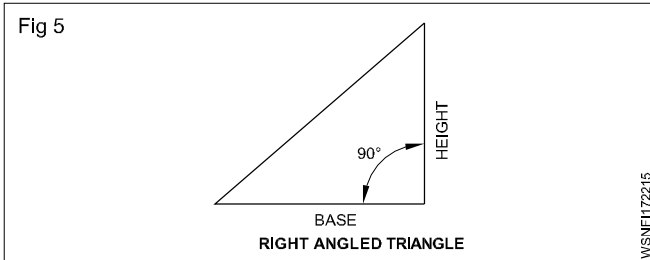
In this triangle, angle between one of two adjacent sides is 90°. Right angle means ninety degrees. That's why right angled triangle means, one of the angles of this triangle is definitely ninety degrees.

Area of right angled triangle

=  $\frac{1}{2}$  x Base x Height

=  $\frac{1}{2}$  bh unit<sup>2</sup>

Hypotenuse =  $\sqrt{\text{Base}^2 + \text{Height}^2}$



Where hypotenuse means, the diagonal or largest length of the side of right angled triangle.

### Examples

- 1 Calculate its area if the base and height of a Right angled triangle are 10 cm and 3.5 cm respectively.

$$\text{Base (b)} = 10 \text{ cm}$$

$$\text{Height (h)} = 3.5 \text{ cm}$$

$$\text{Area (A)} = ?$$

$$A = \frac{1}{2} \times b \times h$$

$$= \frac{1}{2} \times 10 \times 3.5$$

$$= 17.5 \text{ cm}^2$$

- 2 Calculate the base of a triangle having an area of 15 cm<sup>2</sup> and height is 3.5 cm.

$$\text{Area (A)} = 15 \text{ cm}^2$$

$$\text{Height (h)} = 3.5 \text{ cm}$$

$$\text{Base (b)} = ?$$

$$\frac{1}{2} \times b \times h = A$$

$$\frac{1}{2} \times b \times 3.5 = 15$$

$$b = \frac{15 \times 2}{3.5}$$

$$= 8.57 \text{ cm}$$

- 3 Calculate the height of a triangle whose area is 60 cm<sup>2</sup> and base is 10 cm.

$$\text{Area (A)} = 60 \text{ cm}^2$$

$$\text{Base (B)} = 10 \text{ cm}$$

$$\text{Height (h)} = ?$$

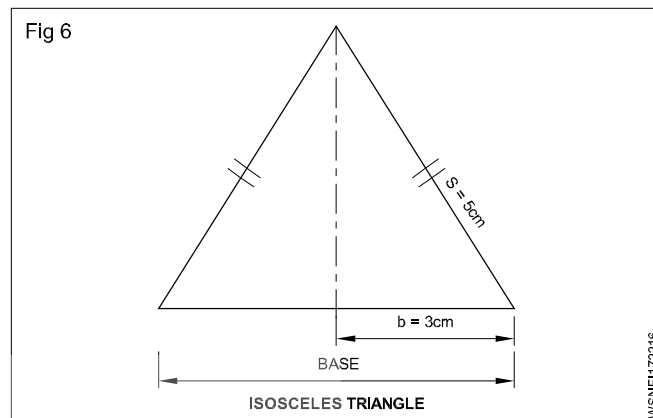
$$\frac{1}{2} \times b \times h = A$$

$$\frac{1}{2} \times 10 \times h = 60$$

$$h = \frac{60 \times 2}{10}$$

$$\text{height } h = 12 \text{ cm}$$

- 4 Find the area of an isosceles triangle whose base is 6 cm long and each of the other two sides 5 cm long.



$$\text{Base (b)} = 6 \text{ cm} = \frac{6}{2} = 3 \text{ cm}$$

$$\text{Equal sides or slant height 's'} = 5 \text{ cm}$$

$$\text{Area (A)} = ?$$

$$A = b \times \sqrt{s^2 - b^2}$$

$$= 3 \times \sqrt{5^2 - 3^2}$$

$$= 3 \times \sqrt{25 - 9}$$

$$= 3 \times \sqrt{16}$$

$$= 3 \times 4$$

$$= 12 \text{ cm}^2$$

or

$$A = \frac{1}{4} b \sqrt{4a^2 - b^2}$$

$$= \frac{1}{4} \times 6 \sqrt{4 \times 5^2 - 6^2}$$

$$= \frac{1}{4} \times 6 \times 8$$

$$= 12 \text{ cm}^2$$

- 5 Find its height if an isosceles triangle has base of 200 mm and its area is 2000 mm<sup>2</sup>.

$$\text{Base} = 200 \text{ mm}$$

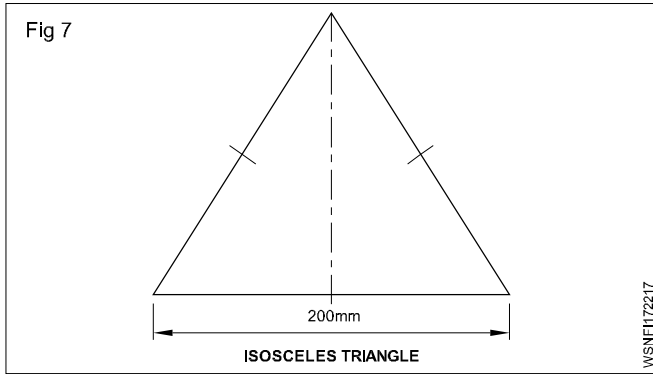
$$\text{Area} = 2000 \text{ mm}^2$$

$$h = ?$$

$$\frac{1}{2} \times b \times h = A$$

$$\frac{1}{2} \times 200 \times h = 2000$$

$$h = \frac{2000 \times 2}{200} = 20 \text{ mm}$$



6 Find the area of an equilateral triangle whose side is 5 cm.

$$\begin{aligned} \text{Area} &= \frac{\sqrt{3}}{4} a^2 \text{ unit}^2 \\ &= \frac{1.732}{4} \times 5 \times 5 \\ &= \mathbf{10.825 \text{ cm}^2} \end{aligned}$$

7 Calculate its perimeter if one side of an equilateral triangle is 55 mm long.

$$\begin{aligned} \text{Side} &= 55 \text{ mm} \\ \text{Perimeter (P)} &= ? \\ P &= 3a \text{ unit} \\ &= 3 \times 55 \\ &= \mathbf{165 \text{ mm}} \end{aligned}$$

8 Find the area of the triangle having its sides are 9cm, 10cm and 12 cm.

$$\begin{aligned} \text{Semi Perimeter} &= \frac{a+b+c}{2} \text{ unit} \\ &= \frac{9+10+12}{2} = \frac{31}{2} \\ &= 15.5 \text{ cm} \\ \text{Area A} &= \sqrt{s(s-a)(s-b)(s-c)} \text{ unit}^2 \\ &= \sqrt{15.5(15.5-9)(15.5-10)(15.5-12)} \\ &= \sqrt{15.5 \times 6.5 \times 5.5 \times 3.5} \\ &= \sqrt{1939.4375} \\ &= \mathbf{44.03 \text{ cm}^2} \end{aligned}$$

9 Find the cost of polishing on both sides of a triangular metal plate has sides 60 cm, 50 cm and 20 cm at the rate of Rs. 1.35 per 100 cm<sup>2</sup>

$$\text{Semi Perimeter} = \frac{a+b+c}{2} \text{ unit}$$

$$= \frac{60+50+20}{2} = \frac{130}{2}$$

$$= 65 \text{ cm}$$

$$\text{Area A} = \sqrt{s(s-a)(s-b)(s-c)} \text{ unit}^2$$

$$= \sqrt{65(65-60)(65-50)(65-20)}$$

$$= \sqrt{65 \times 5 \times 15 \times 45}$$

$$= \mathbf{468.4 \text{ cm}^2}$$

$$\text{Area of polish on both sides} = 2 \times 468.4$$

$$= 936.8 \text{ cm}^2$$

$$\text{Cost of polish per 100 cm}^2 = \text{Rs. } 1.35$$

$$\therefore \text{Cost of polish is } 936.8 \text{ cm}^2 = \frac{936.8}{100} \times 1.35$$

$$= \text{Rs. } 12.65$$

10 Find the area of the right angled triangle with base 20 cm and height 8 cm.

$$\text{Base b} = 20 \text{ cm}$$

$$\text{Equal sides or slant height} = 8 \text{ cm}$$

$$\text{Area (A)} = ?$$

$$\text{Area (A)} = \frac{1}{2} \times \text{base} \times \text{height unit}^2$$

$$= \frac{1}{2} \times 20 \times 8$$

$$= \mathbf{80 \text{ cm}^2}$$

11 Find the area of the right angled triangle if the sides containing the right angle being 10.5 cm and 8.2 cm.

$$\text{Area (A)} = \frac{1}{2} \times \text{base} \times \text{height unit}^2$$

$$= \frac{1}{2} \times 10.5 \times 8.2$$

$$= \mathbf{43.05 \text{ cm}^2}$$

12 Calculate the perpendicular height of the triangle if the area of the right angled triangle is 19.44 m<sup>2</sup> and its one of the adjacent side containing the right angle being 5.4 m.

$$\frac{1}{2} \times \text{base} \times \text{height unit}^2 = \text{Area}$$

$$\frac{1}{2} \times 5.4 \times h = 19.44$$

$$h = \frac{19.44 \times 2}{5.4}$$

$$= 7.2 \text{ m}$$

13 Calculate the base of a right angled triangle having an area of  $15 \text{ cm}^2$ . If its height is  $3.5 \text{ cm}$ .

$$\frac{1}{2} \times \text{base} \times \text{height unit}^2 = \text{Area}$$

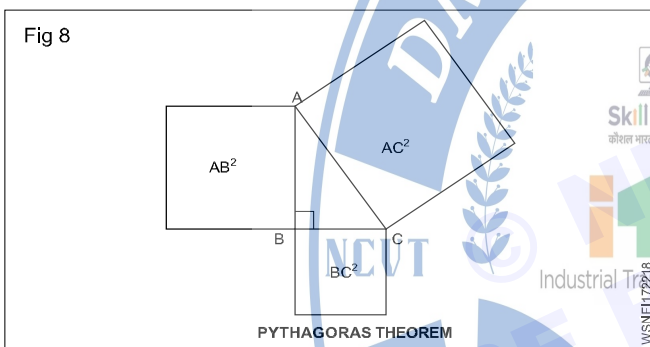
$$\frac{1}{2} \times b \times 3.5 = 15$$

$$b = \frac{15 \times 2}{3.5}$$

$$= 8.57 \text{ cm}$$

### Pythagoras theorem

In a right angled triangle the area of the square drawn with the hypotenuse as the side is equal to the sum of the areas of the squares drawn with the other two sides.



$$\angle B = 90^\circ$$

AC = Hypotenuse

AB & BC = Adjacent sides

As per pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$\therefore AC = \sqrt{AB^2 + BC^2}$$

1 Calculate the hypotenuse of a right angled triangle whose base is  $5 \text{ cm}$  and height is  $12 \text{ cm}$ .

As per pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

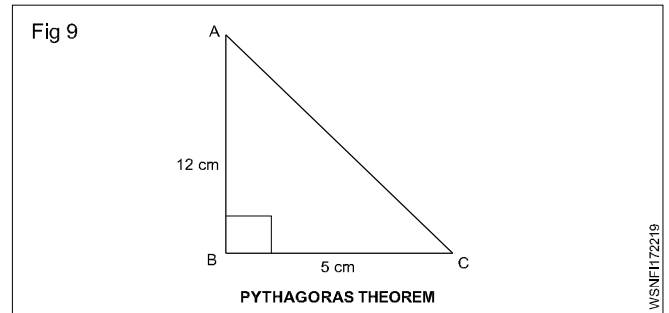
$$= 12^2 + 5^2$$

$$= 144 + 25$$

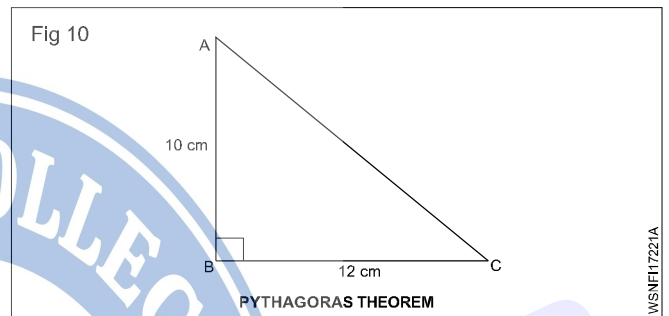
$$= 169$$

$$AC = \sqrt{169}$$

$$= 13 \text{ cm}$$



2 What is the length of the hypotenuse of a right angled triangle, when the sides containing the right angles are  $10 \text{ cm}$  and  $12 \text{ cm}$ .



As per pythagoras theorem,

$$AC^2 = AB^2 + BC^2$$

$$= 10^2 + 12^2$$

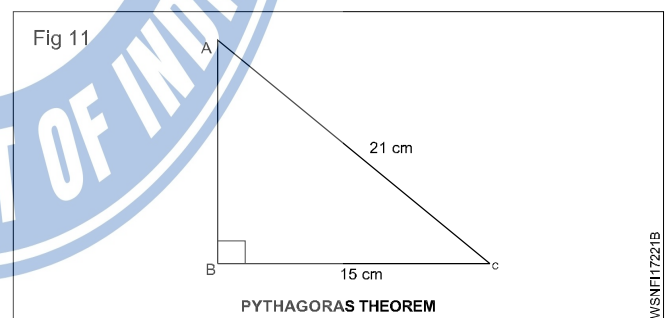
$$= 100 + 144$$

$$= 244$$

$$AC = \sqrt{244}$$

$$= 15.62 \text{ cm}$$

3 Find the height of a right angled triangle whose base is  $15 \text{ cm}$  and hypotenuse is  $21 \text{ cm}$ .



As per pythagoras theorem,

$$AB^2 + BC^2 = AC^2$$

$$AB^2 + 15^2 = 21^2$$

$$AB^2 = 441 - 225$$

$$= 216$$

$$AB = \sqrt{216}$$

$$= 14.7 \text{ cm}$$

Mensuration - Area and perimeter of circle, semi-circle, circular ring, sector of circle, hexagon and ellipse

Circle

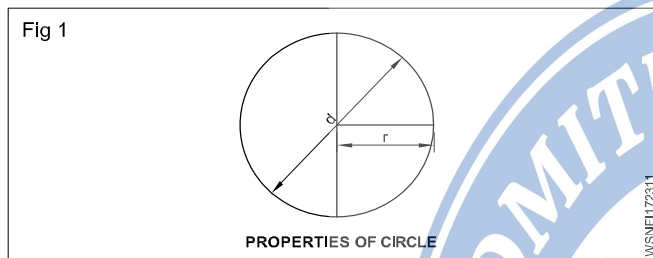
It is the path of a point which is always equal from its centre is called a circle.

r = radius of the circle

d = diameter of the circle

$$\pi = \frac{22}{7} = 3.14$$

Area of the circle =  $\pi r^2$



(or) 
$$= \frac{\pi}{4} d^2 \text{ unit}^2$$

Circumference of the circle  $2\pi r$  (or)  $\pi d$  unit

Examples

1 Find the area of a circle whose radius is 1.54 m. Also find its circumference.

radius r = 1.54 cm

Area A = ?

Circumference C = ?

$$A = \pi r^2 \text{ unit}^2$$

$$= \frac{22}{7} \times 1.54 \times 1.54$$

$$= 7.4536 \text{ m}^2$$

$$C = 2\pi r \text{ unit}$$

$$= 2 \times \frac{22}{7} \times 1.54$$

$$= 9.68 \text{ m}$$

2 Find out the circumference if the area of a circular shape of land is 616 m<sup>2</sup>.

A =  $\pi r^2$  unit<sup>2</sup>

$$r^2 = \frac{616}{\pi}$$

$$= \frac{616 \times 7}{22}$$

$$= 196$$

$$r = \sqrt{196}$$

$$= 14 \text{ m}$$

Circumference =  $2\pi r$  unit

$$= 2 \times \frac{22}{7} \times 14$$

$$= 88 \text{ m}$$

3 Find the side of square into which it can be bent if a wire is in the form of a circle of radius 49 cm.

radius of circle r = 49 cm

side of square = ?

Perimeter of the square = Perimeter of the circle

$$4a = 2\pi r$$

$$4a = 2 \times \frac{22}{7} \times 49$$

$$4a = 308$$

$$a = \frac{308}{4}$$

$$= 77 \text{ cm}$$

4 Find its radius if the difference between the circumference and diameter of a circle is 28 cm.

Circumference - Diameter = 28 cm

$$2\pi r - d = 28$$

$$2\pi r - 2r = 28$$

$$2r(\pi - 1) = 28$$

$$2r \left( \frac{22}{7} - 1 \right) = 28$$

$$2r \left( \frac{22-7}{7} \right) = 28$$

$$2r \times \frac{15}{7} = 28$$

$$r = \frac{28 \times 7}{15 \times 2}$$

$$= 6.53 \text{ cm}$$

5 What is the side of the largest square cut out from a circle of 50 cm dia.?

Diagonal of a square = Diameter of the circle

$$\sqrt{2}a = 50$$

$$a = \frac{50}{\sqrt{2}}$$

$$= \frac{50}{1.414}$$

$$= 35.36 \text{ cm}$$

$$= \frac{1}{2} \times 50 \times 16 \text{ cm}^2$$

$$= 400 \text{ cm}^2$$

$$\text{Area of Semi circle} = \frac{\pi r^2}{2} \text{ unit}^2$$

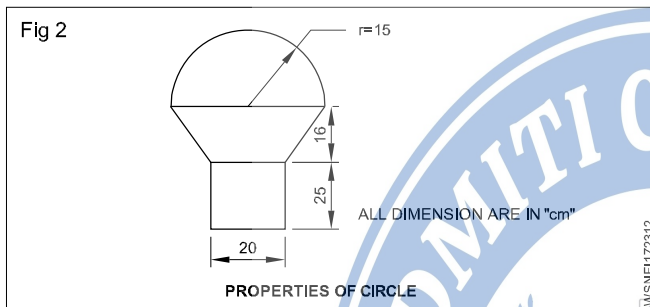
$$= \pi \times 15^2 \times \frac{1}{2} \text{ cm}^2$$

$$= 353.57 \text{ cm}^2$$

$$\text{Total area of the figure} = 500 + 400 + 353.57$$

$$= 1253.57 \text{ cm}^2$$

6 Calculate the area of the figure given below.



$$\begin{aligned} \text{Area of rectangle} &= lb \text{ unit}^2 \\ &= 25 \times 20 \text{ cm}^2 \\ &= 500 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Area of Trapezium} &= \frac{1}{2} \times (a + b) h \\ &= \frac{1}{2} \times (30 + 20) \times 16 \text{ cm}^2 \end{aligned}$$

7 Find the area of remaining steel plate if in a rectangular steel plate 16 cm x 12 cm, there are 6 holes each 4 cm in diameter.

Area of a rectangular plate = length x breadth unit<sup>2</sup>

$$= 16 \times 12$$

$$= 192 \text{ cm}^2$$

$$\text{No. of holes} = 6$$

$$\text{Radius of hole} = 2 \text{ cm}$$

$$\text{Area of 6 holes} = 6 \times \pi r^2 \text{ unit}^2$$

$$= 6 \times \frac{22}{7} \times 2 \times 2 \text{ unit}^2$$

$$= 75.43 \text{ cm}^2$$

$$\text{Area of remaining plate} = 192 - 75.43$$

$$= 116.57 \text{ cm}^2$$

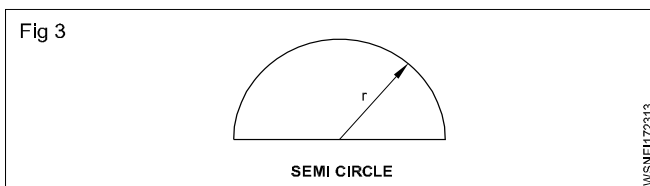
## Semi circle

A semi circle is a sector whose central angle is 180°.

Length of arc of semi circle.

$$\begin{aligned} \text{Length of arc } \ell &= 2\pi r \times \frac{180}{360} \\ &= 2\pi r \times \frac{1}{2} = \pi r \text{ unit} \end{aligned}$$

$$\text{Area of semi circle} = \frac{\pi r^2}{2} \text{ Sq. units}$$



$$\begin{aligned} \text{Perimeter of a semi circle} &= \frac{2\pi r}{2} + 2r \\ &= \pi r + 2r \\ &= r(\pi + 2) \text{ unit} \end{aligned}$$

## Examples

1 Calculate the circumference and area of a semi circle whose radius is 6 cm.

$$\text{radius } r = 6 \text{ cm}$$

$$\text{Area } A = ?$$

$$\text{Circumference } c = ?$$

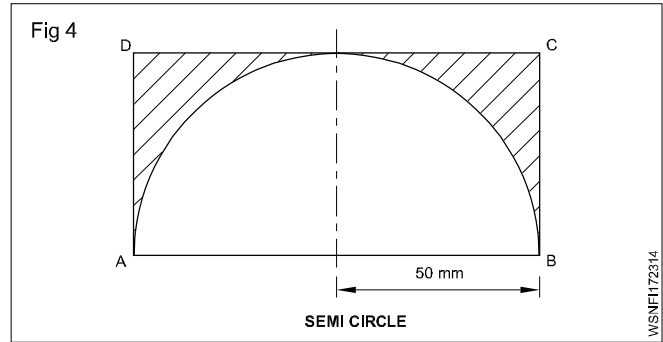
$$A = \frac{\pi r^2}{2} \text{ unit}^2$$

$$= \frac{22}{7} \times \frac{1}{2} \times 6^2$$

$$\text{Area (A)} = \frac{22}{7} \times \frac{1}{2} \times 36$$

$$= \frac{396}{7} = 56.57 \text{ cm}^2$$

$$\begin{aligned} \text{Perimeter of a semicircle} &= 6\left(\frac{22}{7} \times 2\right) \\ &= 6\left(\frac{22+14}{7}\right) \\ &= 6 \times \frac{36}{7} \\ &= \frac{216}{7} \\ &= \mathbf{30.86 \text{ cm}} \end{aligned}$$



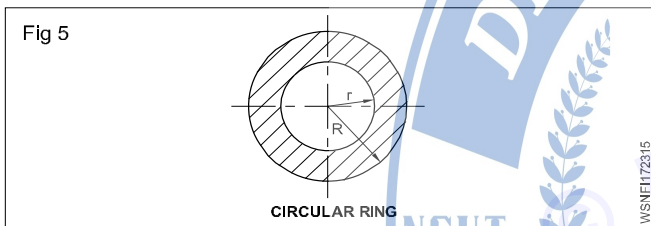
Waste area = Plate area - Area of semi circle

$$\begin{aligned} &= lb - \frac{\pi r^2}{2} \\ &= 100 \times 50 - \frac{22 \times 50 \times 50}{7 \times 2} \\ &= 5000 - 3928.57 \\ &= \mathbf{1071.43 \text{ mm}^2} \end{aligned}$$

2 From the figure given below ABCD is a steel plate, a semi circular plate of radius 50 mm has been prepared by gas cutting. Find the waste area.

Plate length AB = 100 mm  
Breadth BC = 50 mm  
Radius = 50 mm

### Circular ring



R = Outer radius of circular ring

r = Inner radius of circular ring

Area of circular ring =  $\pi (R^2 - r^2)$  unit<sup>2</sup>

or

$A = \pi (R + r) (R - r)$  unit<sup>2</sup>

1 Calculate the area of cross section of pipe having outside dia of 17 cm and inside dia of 14 cm.

Given:

Outer dia of pipe = 17 cm

Outer radius of pipe (R) =  $\frac{17}{2} = 8.5$  cm

Inner dia of pipe = 14 cm

Inner radius of pipe (r) =  $\frac{14}{2} = 7$  cm

To find:

Area of cross section of pipe = ?

Solution:

$$\begin{aligned} \text{Area of cross section of pipe} &= \pi (R + r) (R - r) \text{ unit}^2 \\ &= \pi (8.5 + 7) (8.5 - 7) \\ &= \frac{22}{7} \times 15.5 \times 1.5 \text{ cm}^2 \\ &= \mathbf{73 \text{ cm}^2} \end{aligned}$$

2 Find the distance between the boundaries and the area of the circular ring, if the circumference of two concentric circle are 134 cm and 90 cm.

Given:

Circumference of outer circle = 134 cm

Circumference of inner circle = 90 cm

Distance between the circles = ?

Area of circular ring = ?

Solution:

Circumference of outer circle = 134 cm

$$2\pi R = 134 \text{ cm}$$

$$R = \frac{134}{2\pi} = 21.32 \text{ cm}$$

Circumference of inner circle = 90 cm

$$2\pi r = 90 \text{ cm}$$

$$r = \frac{90}{2\pi} = 14.32 \text{ cm}$$

Distance between the circle = R - r

$$= 21.32 - 14.32 \text{ cm}$$

$$= 7 \text{ cm}$$

Area of circular ring

$$= \pi (R + r) (R - r) \text{ unit}^2$$

$$= \pi (21.32 + 14.32) (21.32 - 14.32) \text{ cm}^2$$

$$= \frac{22}{7} \times 35.64 \times 7 \text{ cm}^2$$

$$= \mathbf{784.08 \text{ cm}^2}$$

3 A wire can be bend in the form of a circle of radius 56 cm. If it is bend in a form of a square, find the side.

Given:

Radius of circle = 56 cm

To find:

Side of square = ?

Solution:

Radius of circle = 56 cm

Circumference of circle =  $2\pi r$  unit =  $2\pi \times 56$  cm

Side of square =  $x$  cm

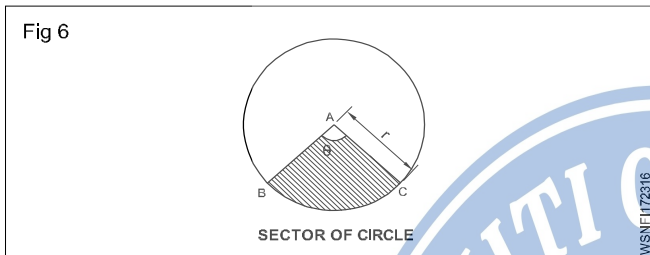
Wire can be bend from the form of round to square

Perimeter of square = circumference of circle

$4 \times a = 352$  cm

$a = \frac{352}{4} = 88$  cm

## Sector of Circle



$\theta$  = Angle of sector of circle

$l$  = Arc length

$r$  = radius

Length of Arc  $l = \frac{\theta}{360^\circ} \times 2\pi r$  unit

Perimeter  $P = 2r + l$  unit

Area =  $\frac{\theta}{360^\circ} \times \pi r^2$  unit<sup>2</sup> (or)  $A = \frac{lr}{2}$  unit<sup>2</sup>

1 Find the perimeter and area of a sector of circle of radius 7 cm and its angle is  $120^\circ$ .

Given:

Angle of sector of circle =  $120^\circ$

Radius = 7 cm

To find:

Perimeter = ? , Area = ?

Solution:

Length of arc ( $l$ ) =  $\frac{\theta}{360^\circ} \times 2\pi r$  unit  
 $= \frac{120}{360} \times 2 \times \frac{22}{7} \times 7$  cm  
 $= 14.67$  cm

Perimeter =  $2r + l$  unit  
 $= 2 \times 7 + 14.67$  cm  
 $= 28.67$  cm

Area =  $\frac{\theta}{360^\circ} \times \pi r^2$  unit<sup>2</sup>

Area =  $\frac{120^\circ}{360^\circ} \times \frac{22}{7} \times 7 \text{ cm}^2 = 51.33 \text{ cm}^2$

2 Find the radius of the circle if the angle is  $60^\circ$  and the area of a sector of a circle is  $144 \text{ cm}^2$ ,

Given:

Area of sector of circle (A) =  $144 \text{ cm}^2$

Angle of sector of circle  $\theta = 60^\circ$

To find:

Radius of circle = ?

Solution:

Area (A) =  $\frac{\theta}{360^\circ} \times \pi r^2$  unit<sup>2</sup>  
 $144 = \frac{60^\circ}{360^\circ} \times \frac{22}{7} \times r^2 \text{ cm}^2$   
 $r^2 = 274.91 \text{ cm}^2$   
 $r = \sqrt{274.91} = 16.58$  cm

3 Find the area of the sector whose angle is  $105^\circ$ , and the perimeter of sector of circle is 18.6 cm.

Given:

Perimeter of a sector of a circle = 18.6 cm

Angle of sector of circle =  $105^\circ$

To find:

Area = ?

Solution:

Length of Arc ( $l$ ) =  $\frac{\theta}{360^\circ} \times 2\pi r$  unit  
 $l = \frac{105^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times r$   
 $= 1.83r$

Perimeter (P) =  $l + 2r$  unit  
 $18.6 = 1.83r + 2r$   
 $3.83r = 18.6$  cm

$r = \frac{18.6}{3.83} = 4.86$  cm

$$\begin{aligned} \text{Area A} &= \frac{\theta}{360^\circ} \times \pi r^2 \text{ unit}^2 \\ &= \frac{105^\circ}{360^\circ} \times \frac{22}{7} \times (4.86) \text{ cm}^2 \\ &= 21.65 \text{ cm}^2 \end{aligned}$$

4 Find the area, if the radius is 12.4 cm and the perimeter of a sector of a circle is 64.8 cm.

Given:

$$\text{Perimeter P} = 64.8 \text{ cm}$$

$$\text{Radius r} = 12.4 \text{ cm}$$

To find:

$$\text{Area A} = ?$$

Solution:

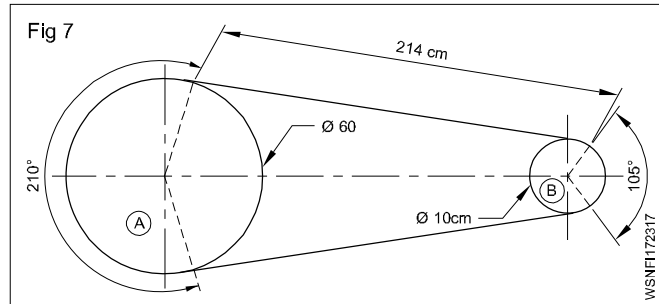
$$\text{Perimeter P} = l + 2r \text{ unit}$$

$$l = P - 2r \text{ unit}$$

$$\begin{aligned} &= 64.8 - 2(12.4) \text{ cm} \\ &= 64.8 - 24.8 = 40 \text{ cm} \end{aligned}$$

$$\begin{aligned} \text{Area A} &= \frac{lr}{2} \text{ unit}^2 = \frac{40 \times 12.4}{2} \\ &= 248 \text{ cm}^2 \end{aligned}$$

5 Find out the length of the belt, if the arrangement of a belt is shown in the figure below.



Solution:

$$\text{Length } l_A = \frac{\theta}{360^\circ} \times 2\pi r \text{ unit}$$

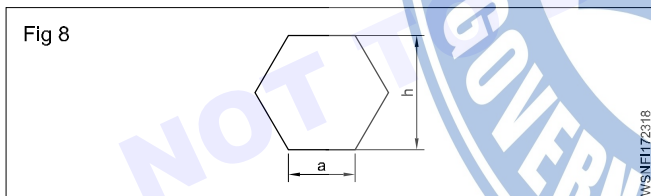
$$= \frac{210^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 30 = 110 \text{ cm}$$

$$\text{Length } l_B = \frac{\theta}{360^\circ} \times 2\pi r \text{ unit}$$

$$= \frac{105^\circ}{360^\circ} \times 2 \times \frac{22}{7} \times 5 = 91.7 \text{ cm}$$

$$\begin{aligned} &= l_A + l_B + 2 \times 214 \text{ cm} \\ &= 110 + 9.17 + 428 \text{ cm} \\ &= 547.17 \text{ cm} \end{aligned}$$

## Hexagon



Side = a unit

Perimeter P = 6a unit

Area A =  $6 \times \frac{\sqrt{3}}{4} \times a^2$  units<sup>2</sup> (Area of 6 equilateral triangle)

DAF (Distance Across Flats) =  $\sqrt{3} \times a$  unit

DAC (Distance Across Corners) = 2 x a unit

1 Find out the perimeter, area, DAF and DAC of a regular hexagon whose side is 2cm.

(DAF - Distance Across Flats)

(DAC - Distance Across Corners)

Given: Side of hexagon (a) = 2cm

To Find: P = ?, A = ?, DAF = ?, DAC = ?

Solution:

$$\begin{aligned} \text{Perimeter of hexagon (P)} &= 6a \text{ unit} \\ &= 6a \text{ unit} = 6 \times 2 \text{ cm} = 12 \text{ cm} \end{aligned}$$

$$\text{Area of hexagon A} = 6 \times \frac{\sqrt{3}}{4} \times a^2 \text{ unit}^2$$

$$\begin{aligned} &= 6 \times \frac{1.732}{4} \times 2^2 \\ &= 10.392 \text{ cm}^2 \end{aligned}$$

DAF (Distance Across

Flats) =  $\sqrt{3} \times a$  unit

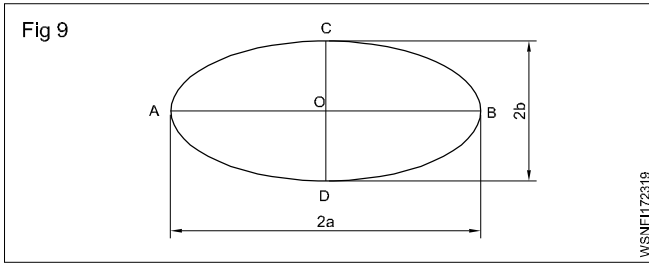
$$\begin{aligned} &= \sqrt{3} \times 2 = 1.732 \times 2 \\ &= 3.464 \text{ cm} \end{aligned}$$

DAC (Distance Across

Corners) = 2 x a unit

$$= 2 \times 2 = 4 \text{ cm}$$

## Ellipse



Major axis  $AB = 2a$

Half of Major axis  $OB = a$ ,

Minor axis  $CD = 2b$

Half of Minor axis  $OC = b$

Area of ellipse  $A = \pi \times a \times b \text{ unit}^2$

Perimeter of ellipse  $P = 2\pi \sqrt{\frac{(a^2 + b^2)}{2}} \text{ unit}$

- Find its area and perimeter, if the major and minor axis of an ellipse are 12 cm and 8 cm respectively.

Solution:

$$\begin{aligned} \text{Major axis } 2a &= 12 \text{ cm} \\ a &= \frac{12}{2} = 6 \text{ cm} \end{aligned}$$

$$\text{Minor axis } 2b = 8 \text{ cm}$$

$$b = \frac{8}{2} = 4 \text{ cm}$$

$$\text{Area } A = \pi \times a \times b \text{ unit}^2$$

$$= \frac{22}{7} \times 6 \times 4 \text{ cm}^2$$

$$= 75.43 \text{ cm}^2$$

$$\text{Perimeter (P)} = 2\pi \sqrt{\frac{(a^2 + b^2)}{2}} \text{ unit}$$

$$= 2 \times \frac{22}{7} \sqrt{\frac{(6^2 + 4^2)}{2}} \text{ unit}$$

$$= 2 \times \frac{22}{7} \sqrt{\frac{36 + 16}{2}} \text{ unit}$$

$$= 2 \times \frac{22}{7} \times \sqrt{26}$$

$$= 2 \times \frac{22}{7} \times 5.1 = 32.06 \text{ cm}$$

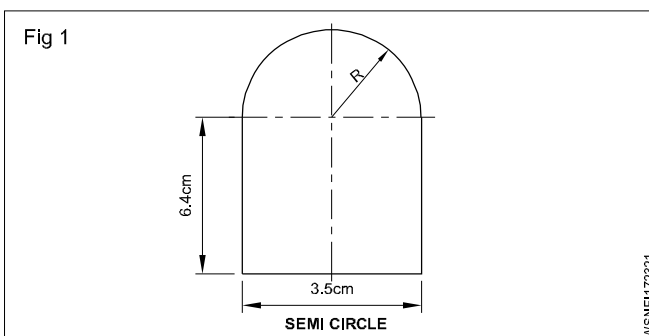
## Assignment

### Circle

- Find the circumference and area of a circle whose radius is 10.00 metre.
- Find its diameter if the area of a circle is  $330 \text{ cm}^2$ .
- Find its area if the circumference of a circle is 50 cm.
- Find out the area and circumference of a circle of diameter is 50 cm.
- Find its area if the circumference of a circle is 44 cm.

### Semi circle

- Calculate the circumference and area of semi circle whose radius is 14 cm.
- Find area of the given figure.



### Circular ring

- Find out area of a ring washer, whose inner radius and outer radius are 13 cm and 15 cm respectively.
- Find the area of a ring portion of a washer whose outer dis is 30 m and inner dis is 20 m. Also calculate the difference between the circumference of circles.

### Sector of circle

- Find the perimeter and area of a sector of a circle of radius 5cm and its angle is  $96^\circ$ .
- Find the radius of the circle if the angle is  $90^\circ$  and the area of sector of a circle is  $196 \text{ cm}^2$ .

### Hexagon

- Find out the Area, perimeter, DAF, and DAC of hexagon of side 4cm.
- Find the area of cross section of a regular hexagon rod whose side is 7.5 cm.

### Ellipse

- Find the area of the biggest ellipse that can be inscribed in a rectangle of length 18 cm and breadth 12 cm. Also calculate its perimeter.
- How much fencing will be required to enclose an elliptical plot of ground the axes of the ellipse being 200 and 170 meter respectively.

Mensuration - Surface area and volume of solids - Cube, cuboid, cylinder, sphere and hollow cylinder

**Cube**

All sides of cube are same i.e length, breadth and height have same value. It is bounded by six equal square faces.

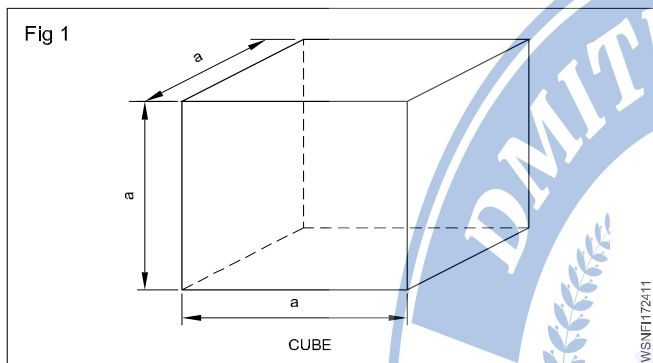
Volume of cube = side x side x side  
 =  $a^3 \text{ unit}^3$

Lateral surface area =  $4a^2 \text{ unit}^2$

Total surface area = 6 x side x side  
 =  $6a^2 \text{ unit}^2$

$\sqrt{3} = 1.732$

Diagonal d =  $\sqrt{3}a$



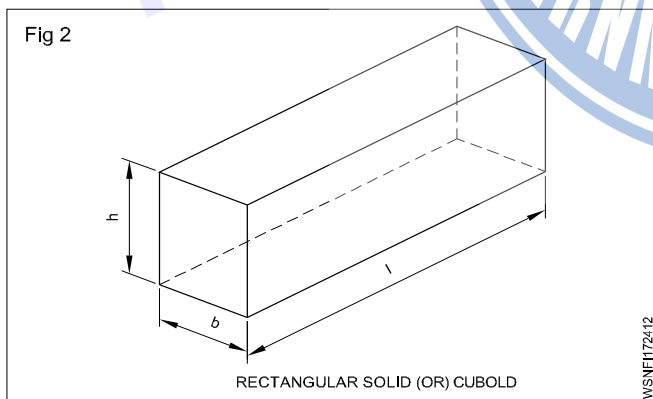
**Rectangular solid (or) cuboid**

Rectangular solid is bounded by six rectangular surfaces and opposite surfaces are equal and parallel to each other.

Volume of rectangular solid = Length x breadth x height  
 =  $l . b . h \text{ unit}^3$

Lateral surface area =  $2h(l+b) \text{ unit}^2$

Total surface area =  $2lb + 2bh + 2hl$   
 =  $2(lb+bh+hl) \text{ unit}^2$

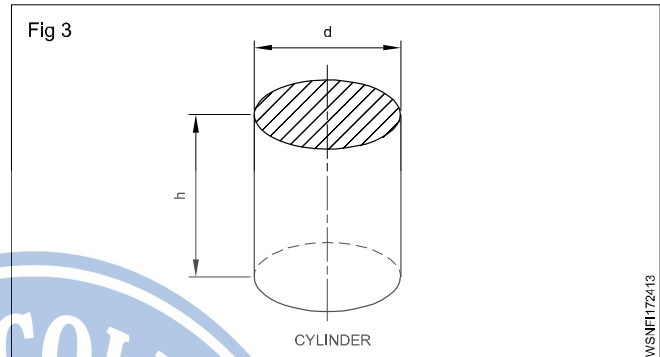


l = length, b = breadth and h = height

**Cylinder**

This is a prism whose top and bottom surfaces are equal and circular.

Volume of cylinder =  $\pi r^2 h$  or  $\frac{\pi}{4} d^2 h$



Curved area of cylinder =  $2\pi rh \text{ unit}^2$

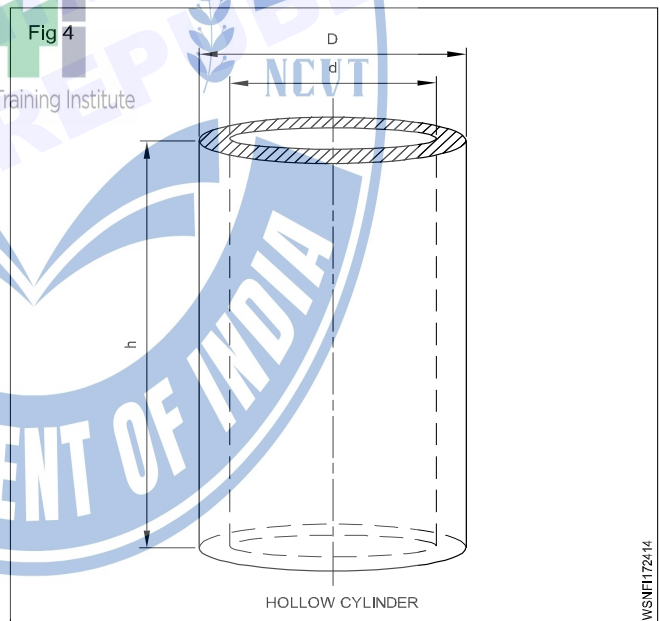
Total surface area of cylinder =  $2\pi r(h+r) \text{ unit}^2$

r = Radius of base, d = Diameter of base

h = Height of cylinder

**Hollow cylinder**

Hollow means empty space. In hollow cylinder there is an empty place. Water pipe is an example of hollow cylinder.



Volume of hollow cylinder =  $\pi (R^2 - r^2) h$  (or)  
 =  $\pi (R + r) (R - r) h$  (or)  
 =  $\frac{\pi}{4} (D^2 - d^2) h \text{ unit}^3$   
 =  $\frac{\pi}{4} (D + d)(D - d) h$

Total surface area of hollow cylinder =

Inner + outer curved area + area of top and bottom circular part

$\therefore \text{TSA} : 2\pi Rh + 2\pi rh + 2\pi(R^2 - r^2)$

R = outer radius

r = inner radius

D = outer diameter

d = inner diameter

h = height of cylinder

t = thickness

$$\text{Mean dia} = \frac{D-d}{2}$$

If thickness given then:

Volume of hollow cylinder =  $\pi \times$  mean dia  $\times$  thickness  $\times$  height

### Finding out volumes of solids

The space occupied by a body is known its volume. The volume of a body indicates the capacity to hold substance in it.

The general form of Lateral surface area Total surface area and Volume is :

**Lateral surface area** = perimeter of the base  $\times$  height

**Total surface area** = LSA + 2 (base area )

**Volume** = Area of base  $\times$  height

Important and commonly used solids are described below one after another:

### Cube

1 Find the diagonal, lateral surface area, total surface area and volume of a cube of side 4.5 cm.

side a = 4.5 cm

diagonal d =  $\sqrt{3}a$  unit  
= 1.732  $\times$  4.5  
= 7.794 cm

L.S.A =  $4a^2$  unit<sup>2</sup>  
= 4  $\times$  4.5  $\times$  4.5  
= 81 cm<sup>2</sup>

T.S.A =  $6a^2$  unit  
= 6  $\times$  4.5  $\times$  4.5  
= 121.5 cm<sup>2</sup>

V =  $a^3$  unit<sup>3</sup>  
= 4.5  $\times$  4.5  $\times$  4.5  
= **91.125 cc.**

2 Calculate volume of a cube where side is 9 cm

a = 9 cm  
V = ?  
V =  $a^3$   
= 9  $\times$  9  $\times$  9  
= **729 cm<sup>3</sup>**

3 Find out side of the cube if a cube has volume of 3375cm<sup>3</sup>.

V = 3375 cm<sup>3</sup>  
a = ?  
 $a^3 = 3375$   
a =  $\sqrt[3]{3375}$   
=  $\sqrt{3 \times 3 \times 3 \times 5 \times 5 \times 5}$   
= 3  $\times$  5  
= **15 cm**

4 Find the side of a cube, if its surface area is 216 cm<sup>2</sup>

Surface area = T.S.A = 216 cm<sup>2</sup>

$6a^2 = 216$

$a^2 = \frac{216}{6}$

= 36

a =  $\sqrt{36}$

= 6 cm

5 Find the side of the square tank, if its height is 2 metre and has the capacity to hold 50,000 litre of water.

Height of square shape tank (h) = 2 m

Capacity = 50,000 litre

1000 litre = 1 m<sup>3</sup>

50,000 Litre =  $\frac{50000}{1000}$

= 50 m<sup>3</sup>

Capacity of tank = 50 m<sup>3</sup>

$a^2 \times h = 50$

$a^2 \times 2 = 50$

$a^2 = \frac{50}{2} = 25 \text{ m}^2$

a =  $\sqrt{25} = 5 \text{ m}$

**Side of the square tank = 5 m**

### Rectangular Solid (or) Cuboid

1 Find its volume and T.S.A if a tank is 20 m long, 15 m broad and 12 m high.

l = 20m

b = 15 m

h = 12 m

v = ?

T.S.A = ?

Volume v = lbh unit<sup>3</sup>

$$= 20 \times 15 \times 12$$

$$= 3600 \text{ m}^3$$

$$\text{T.S.A} = 2(lb + bh + hl) \text{ unit}^2$$

$$= 2((20 \times 15) + (15 \times 12) + (20 \times 12))$$

$$= 2(300 + 180 + 240)$$

$$= \mathbf{1440 \text{ m}^2}$$

- 2 Find out its height if the cross section is 260 mm length and 180 mm wide rectangular and the capacity of a fuel tank is 10500 cm<sup>3</sup>.

$$l = 260\text{mm} = 26 \text{ cm}$$

$$b = 180 \text{ mm} = 18 \text{ cm}$$

$$v = 10500 \text{ cm}^3$$

$$h = ?$$

$$l.b.h = \text{volume}$$

$$26 \times 18 \times h = 10500$$

$$h = \frac{10500}{26 \times 18}$$

$$= \mathbf{22.44 \text{ cm}}$$

- 3 How many litres of water it can store if a water tank has the following dimensions length = 1 metre, width = 0.8 metre and height = 1.2 metre?

$$\text{Volume} = l \times b \times h \text{ unit}^3$$

$$= 1 \times 0.8 \times 1.2$$

$$= 0.96 \text{ m}^3$$

$$= 0.96 \times 1000$$

**= 960 litres of water can store in the tank.**

- 4 Find its volume if the base of a prism is a rectangle having 5m length, 4m breadth and the height of the prism is 15m.

The base of prism is rectangle

$$\text{Area of base} = \text{length} \times \text{breadth}$$

$$= 5 \times 4$$

$$= 20 \text{ square m}$$

$$\text{Volume of prism} = \text{Area of base} \times \text{Height}$$

$$= 20 \times 15$$

$$= \mathbf{300 \text{ cm}^3}$$

### Cylinder

- 1 Find the volume and total surface are of a cylinder having 9cm diameter and 15 cm height.

$$\text{diameter} = 9 \text{ cm}$$

$$\text{radius } r = 4.5 \text{ cm}$$

$$\text{height } h = 15 \text{ cm}$$

$$\text{Volume } V = ?$$

$$\text{T.S.A} = ?$$

$$V = \pi r^2 h \text{ unit}^3$$

$$= \frac{22}{7} \times 4.5 \times 4.5 \times 15$$

$$= 954.4 \text{ cm}^3$$

T.S.A

$$= 2\pi r(h+r) \text{ unit}^2$$

$$= 2 \times \frac{22}{7} \times 4.5 (15 + 4.5)$$

$$= 2 \times \frac{22}{7} \times 4.5 \times 19.5$$

$$= \mathbf{551.4 \text{ cm}^2}$$

- 2 Calculate the radius if the curved surface area of a cylindrical roller is 48π cm<sup>2</sup> and the roller is 10 cm long

$$\text{C.S.A} = 48\pi \text{ cm}^2$$

$$\text{length} = 10 \text{ cm}$$

$$\text{radius} = ?$$

$$2\pi rh = 48\pi$$

$$2 \times \pi \times r \times 10 = 48\pi$$

$$r = \frac{48 \times \pi}{2 \times \pi \times 10}$$

$$= \mathbf{2.4 \text{ cm}}$$

- 3 Find its radius if the volume of a cylinder is 5544 cm<sup>3</sup> and its height is 16 cm.

$$\pi r^2 h = v$$

$$3.14 \times r^2 \times 16 = 5544$$

$$r^2 = \frac{5544}{3.14 \times 16}$$

$$r^2 = \frac{5544}{50.24}$$

$$= 110.35$$

$$r = \sqrt{110.35}$$

$$= \mathbf{10.5 \text{ cm}}$$

- 4 Find the diameter of the tank if the volume of a circular tank is 68.46 m<sup>3</sup>, its height is 2 m.

$$\pi r^2 h = 68.46$$

$$r^2 = \frac{68.46}{3.14 \times 2}$$

$$r^2 = 10.9$$

$$r = \sqrt{10.9}$$

$$= 3.3 \text{ m}$$

$$\text{diameter} = 2r$$

$$= 2 \times 3.3$$

$$= \mathbf{6.6 \text{ m}}$$

- 5 A cylindrical vessel is to be made of 3 metre long and 1.9994 metre diameter. Calculate its surface area, if it is in a closed form on one end.

$$\begin{aligned} h &= 3\text{ m} \\ d &= 1.9994\text{ m} \\ r &= 0.9997\text{ m} \end{aligned}$$

$$\begin{aligned} \text{T.S.A} &= \text{C.S.A} + \text{Base area} \\ &= 2\pi rh + \pi r^2 \\ &= \left(2 \times \frac{22}{7} \times 0.9997 \times 3\right) + \left(\frac{22}{7} \times 0.9997^2\right) \\ &= 18.85 + 3.14 \\ &= \mathbf{21.99\text{ m}^2} \end{aligned}$$

- 6 How many litres of water a cylinder of radius 75 cm and height 100 cm can hold.

$$\begin{aligned} V &= \pi r^2 h \text{ unit}^3 \\ &= 3.142 \times 75 \times 75 \times 100 \\ &= 1767375 \text{ cm}^3 \\ &= \frac{1767375}{1000} \quad [1000 \text{ cc} = 1 \text{ litre}] \\ &= \mathbf{1767.375 \text{ litres.}} \end{aligned}$$

- 7 Calculate the height of cylindrical tin if a closed rectangular box 40 cm long, 30 cm wide and 25 cm deep has the same volume as that of cylinder tin of radius 17.5 cm.

$$\begin{aligned} \text{Volume of cylinder} &= \text{Volume of rectangular box} \\ \pi r^2 h &= l \times b \times h \\ \frac{22}{7} \times 17.5 \times 17.5 \times h &= 40 \times 30 \times 25 \\ h &= \frac{40 \times 30 \times 25 \times 7}{22 \times 17.5 \times 17.5} \\ &= \frac{210000}{6737.5} \\ &= \mathbf{31.17 \text{ cm}} \end{aligned}$$

- 8 An oxygen cylinder is 15 cm in diameter and 100 cm in length. It is filled with gas under pressure so that every  $\text{cm}^3$  of the cylinder contains 120  $\text{cm}^3$  of gas. How much cc of oxygen does this hold?

$$\begin{aligned} \text{Volume of cylinder} &= \pi r^2 h \text{ unit}^3 \\ &= \frac{22}{7} \times 7.5 \times 7.5 \times 100 \\ &= 17678.57 \text{ cm}^3 \end{aligned}$$

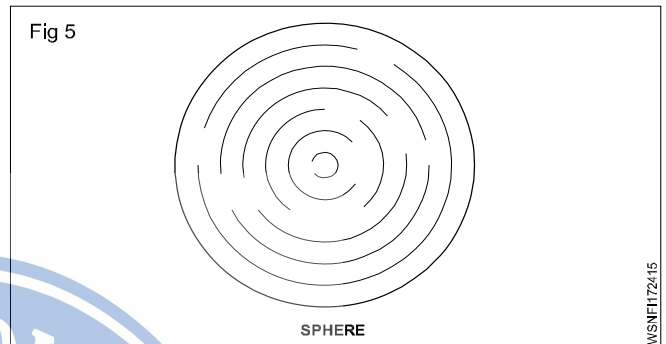
Gas contain in 1  $\text{cm}^3 = 120 \text{ cm}^3$  of gas

$$\begin{aligned} \text{Gas contain in } 17678.57 \text{ cm}^3 &= 17678.57 \times 120 \\ &= 2121428 \text{ cm}^3 \end{aligned}$$

**Volume of oxygen = 2121428 cc.**

## Sphere

Sphere is a solid circular body.



$$\text{Volume of sphere} = \frac{4}{3} \pi r^3 \quad \text{or}$$

$$= \frac{\pi}{6} d^3 \text{ unit}^3$$

$$\text{Total surface area of sphere} = 4\pi r^2 \text{ unit}^2$$

Where r = Radius of sphere

d = Diametre of sphere

$$\text{Radius} = \frac{1}{2} \text{ of diameter}$$

- 1 Find the volume and surface area of a sphere of 3 cm radius.

$$\begin{aligned} V &= \frac{4}{3} \pi r^3 \text{ unit}^3 \\ &= \frac{4 \times 22 \times 3 \times 3 \times 3}{3 \times 7} \end{aligned}$$

$$= 113.1 \text{ cm}^3$$

$$\text{Total Surface Area} = 4\pi r^2 \text{ unit}^2$$

$$= 4 \times \frac{22}{7} \times 3 \times 3$$

$$= \mathbf{113.1 \text{ cm}^2}$$

- 2 Find the diameter of sphere having volume of 15625 cc.

$$\frac{4}{3} \pi r^3 = \text{Volume}$$

$$\frac{4}{3} \times \frac{22}{7} \times r^3 = 15625$$

$$r^3 = \frac{15625 \times 3 \times 7}{4 \times 22}$$

$$= \frac{328125}{88}$$

$$= 3728.69$$

$$r = \sqrt[3]{3728.69}$$

$$= 15.51 \text{ cm}$$

$$\text{diameter} = 2 \times \text{radius}$$

$$= 2 \times 15.51$$

$$= 31.02 \text{ cm}$$

$$\begin{aligned} \text{1st ball } d_1 &= 3 \text{ cm,} \\ r_1 &= 1.5 \text{ cm} \\ \text{2nd ball } d_2 &= 4 \text{ cm,} \\ r_2 &= 2 \text{ cm} \\ \text{3rd ball } d_3 &= 5 \text{ cm,} \\ r_3 &= 2.5 \text{ cm} \end{aligned}$$

Diameter of new ball = ?

Volume of new ball = Volume of 3 spherical balls

$$\frac{4}{3} \pi r^3 = \frac{4}{3} \pi r_1^3 + \frac{4}{3} \pi r_2^3 + \frac{4}{3} \pi r_3^3$$

$$\frac{4}{3} \pi r^3 = \frac{4}{3} \pi (1.5^3 + 2^3 + 2.5^3)$$

$$r^3 = 3.375 + 8 + 15.625$$

$$r^3 = 27$$

$$r = \sqrt[3]{27}$$

$$r = \sqrt[3]{3 \times 3 \times 3}$$

$$r = 3 \text{ cm}$$

$$\text{Diameter of the ball} = 2 \times r$$

$$= 2 \times 3$$

$$= 6 \text{ cm}$$

- 3 How many spherical balls of 1 cm radius can be made from a sphere of 32 cm diameter.

No. of balls x volume of small sphere = Volume of bigger sphere

$$N \times \frac{4}{3} \pi r^3 = \frac{4}{3} \pi R^3$$

$$N \times \frac{4}{3} \pi \times 1^3 = \frac{4}{3} \pi \times 16^3$$

$$N = 16 \times 16 \times 16$$

$$= 4096 \text{ balls}$$

- 4 Three brass balls of diameters 3 cm, 4 cm and 5 cm are melted and make into one solid ball, if there is no wastage. Find the diameter of the solid ball.

## Assignment

### Cube

- Find the diagonal, lateral surface area, total surface area and volume of cube, whose side is 15 cm.
- Find the volume of 10 cubes where each side is 5 cm.
- Find its volume if a solid cube has each of its sides 60 mm long.
- What is its side if the total surface area of a cube is 384 m<sup>2</sup>.

### Cuboid

- Find the volume of the tank in m<sup>3</sup>, if the length is 60 m, breadth 40 m and height 20 m.
- Find the volume of a C.I. casting of a rectangular block having 25 cm x 20 cm x 8 cm size.
- Calculate the total surface area of a box whose length, width and height are 120 cm, 50 cm and 60 cm respectively.
- Find the volume of the sheet if a brass sheet is of 25 cm square and 0.4 cm thick.

### Cylinder

- Find the curved surface area of cylinder whose diameter is 18 cm and height 34 cm?
- Find the total surface area of cylinder whose diameter is 24 cm and height 40 cm?
- Find out the volume of cylinder whose base is 10 cm radius and height is 40 cm?

### Sphere

- Find the volume of sphere having diameter 3.5cm?
- Find the total surface area of a sphere having radius 1.75 cm?
- How many spherical balls of 1 cm radius can be made from a sphere of 16 cm diameter.
- Three balls of diameter 2m, 4cm and 6 cm are melted and made into one solid ball. If there is no wastage, find the diameter of solid ball.

Mensuration - Finding the lateral surface area, total surface area and capacity in litres of hexagonal, conical and cylindrical shaped vessels

**Hexagonal bar**

Volume of Hexagonal bar = Area of hexagonal x height

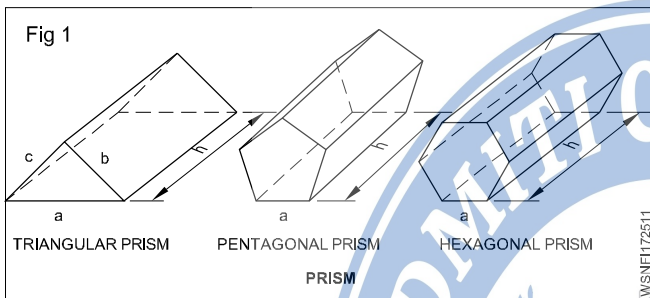
Lateral surface area of hexagonal bar

$$= 6 \times \text{side of hexagon} \times \text{length of the bar}$$

or = 3.464 x length of the bar x flat of hexagon

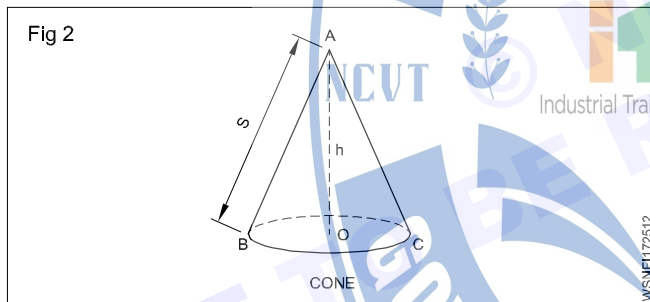
Total surface area of hexagonal bar

$$= \text{lateral surface area} + (2 \times \text{area of hexagon})$$



**Cone**

Cone is a pyramid with a circular base.



$$\text{Volume of cone} = \frac{1}{3} \pi r^2 h$$

$$\text{or} = \frac{\pi}{12} d^2 h$$

Curved area =  $\pi r s$

Total surface area =  $\pi r(s+r)$

Where r = radius of base

d = diameter of base

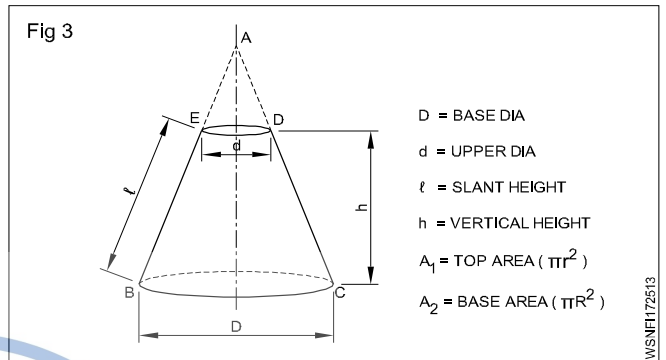
h = vertical height of cone

$$s = \text{slant height} \sqrt{r^2 + h^2}$$

**Frustum of a cone**

When a cone is cut by a plane parallel to the base, and upper part is removed, the formation appears, is termed as frustum of a cone. Buckets, oil cans etc. are such frustums in shape.

$$\text{L.S.A} = \pi l (R + r) \text{ unit}^2$$



$$\text{TSA} = \pi l (R + r) + A_1 + A_2 \text{ unit}^2$$

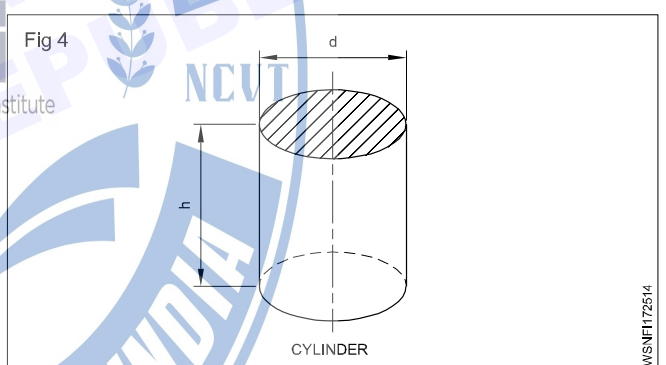
$$V = \frac{\pi}{3} h (R^2 + Rr + r^2) \text{ unit}^3$$

[A<sub>1</sub> = Top area ; A<sub>2</sub> = Bottom area]

**Cylinder**

This is a prism whose top and bottom surfaces are equal and circular.

$$\text{Volume of cylinder} = \pi r^2 h \text{ or } \frac{\pi}{4} d^2 h$$



Curved area of cylinder =  $2\pi r h$

Total surface area of cylinder =  $2\pi r(h+r)$

r = Radius of base, d = Diameter of base

h = Height of cylinder

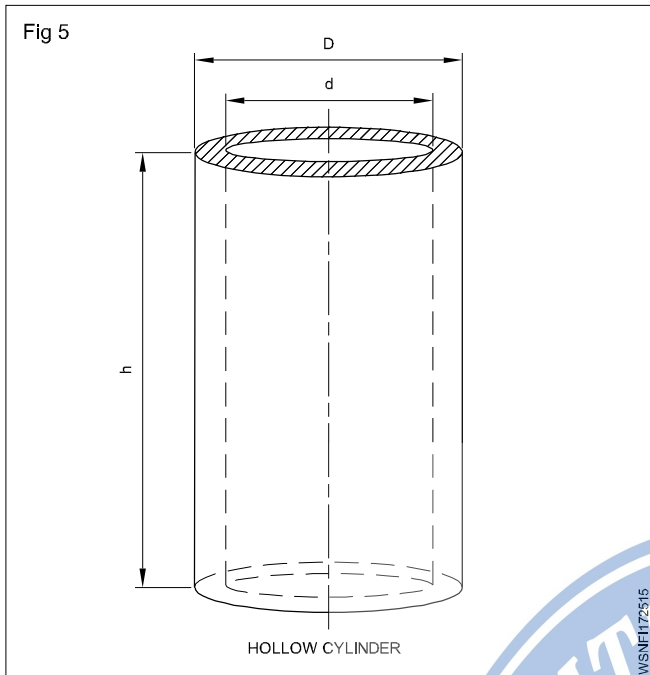
**Hollow cylinder**

Hollow means empty space. In hollow cylinder there is an empty place. Water pipe is an example of hollow cylinder.

$$\begin{aligned} \text{Volume of hollow cylinder} &= \pi (R^2 - r^2) h \text{ (or)} \\ &= \pi (R + r) (R - r) h \text{ (or)} \\ &= \frac{\pi}{4} (D^2 - d^2) h \\ &= \frac{\pi}{4} (D + d)(D - d) h \end{aligned}$$

Total surface area of hollow cylinder =

Inner + outer curved area + area of top and bottom circular part



$$\therefore \text{TSA} : 2\pi Rh + 2\pi rh + 2\pi(R^2 - r^2)$$

R = outer radius

r = inner radius

D = outer diameter

d = inner diameter

h = height of cylinder

t = thickness

$$\text{Mean dia} = \frac{D - d}{2}$$

If thickness given then:

Volume of hollow cylinder =  $\pi \times \text{mean dia} \times \text{thickness} \times \text{height}$

### Example

1 Find the volume of an hexagonal prism having its side 20 cm and height 200 cm.

Side of hexagonal prism (a) = 20 cm

Height (h) = 200 cm

Volume (V) = Base side area x Height

$$= 6 \times \frac{\sqrt{3}}{4} \times a^2 \times h$$

$$= 6 \times \frac{\sqrt{3}}{4} \times 20 \times 20 \times 200$$

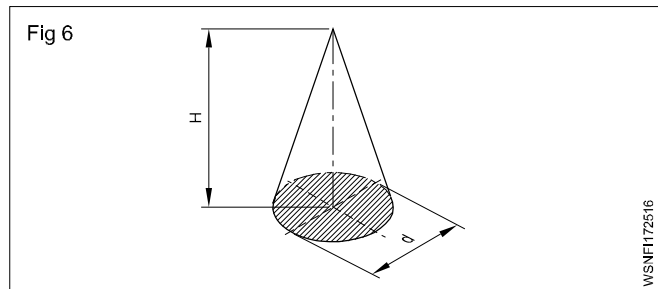
$$= 1,20,000 \times \sqrt{3}$$

$$= 1,20,000 \times 1.732$$

$$= 2,07,840 \text{ cm}^3$$

Volume of the hexagonal prism = 2,07,840 cm<sup>3</sup>

2 Calculate the height. Also find the lateral surface area if a cone has a base diameter of 210 mm and its volume is 3056 cm<sup>3</sup>.



$$\text{Volume of a cone} = \frac{1}{3} \times \text{Area of base} \times \text{height}$$

$$3056 \text{ cm}^3 = \frac{1}{3} \times 0.785 \times 210^2 \text{ mm}^2 \times H$$

$$H = \frac{3056 \times 3 \times 1000 \text{ mm}^3}{0.785 \times 210^2 \text{ mm}^2} = 264.82 \text{ mm}$$

$$L = \text{Slant height} = \sqrt{264.82^2 + 105^2} = 284.9 \text{ mm}$$

$$\begin{aligned} \text{Lateral surface area} &= \frac{1}{2} \pi \times 210 \times 284.9 \text{ mm}^2 \\ &= 94017 \text{ mm}^2 = 940.17 \text{ cm}^2 \end{aligned}$$

3 Determine its diameter in mm if the height of a rod of 1.6 metres and its volume is 1.017 metre<sup>3</sup>.

$$1.017 \text{ m}^3 = 0.785d^2 \times 1.6 \text{ metres}$$

$$0.785d^2 = \frac{1.017}{1.6} \text{ m}^2$$

$$d^2 = \frac{1.017}{1.6 \times 0.785} \text{ m}^2$$

$$= \frac{1.017}{1.6 \times 785} \text{ m}^2$$

$$d = \sqrt{\frac{10170}{16 \times 785}} \text{ metre}$$

$$= \sqrt{\frac{10170}{12560}}$$

$$= \sqrt{0.8097}$$

$$= 0.8998$$

$$= 899.8 \text{ mm}$$