

Trigonometry - Measurement of angles

Introduction:

Trigonometry is the branch of mathematics which deals with the study of measurement and relationship of the three sides and three angles of a triangle.

Units:

Measurement of Angles

There are three systems of measuring the angle:

(i) Sexagesimal System

This is called British System. In this system, one right angle is divided into 90 equal parts which are called degrees. Each part is divided into 60 parts which are called minutes. Each minute is divided into 60 parts which are called seconds. The parts so divided respectively are called:

One degree (1°), one minute (1') and one second (1")

It means 1 right angle = 90° (90 degrees)

1 degree (1°) = 60' (60 minutes)

1 minute (1') = 60" (60 seconds)

In Trigonometry, mostly this system is used.

(ii) Centesimal System

This is called French System. In this system, the right angle is divided into 100 equal parts which are called grades. Each grade is divided into 100 minutes and each minute is divided into 100 seconds.

Parts so divided are respectively called:

One grade (1 g), one minute (1'), one second (1").

It means 1 right angle = 100 grades (100g)

1 grade (1 g) = 100 minutes (100')

1 minute (1') = 100 seconds (100")

90° = 100g (because each is a right angle)

This system is easier than Sexagesimal System. But to use this system many other systems will have to be devised that is why this system is not used.

(iii) Circular System

In this system, the unit of measuring angles is radian. It is that angle which is formed at the centre and is formed of an arc of length equal to radius in a circle.

There is one constant ratio between the circumference and dia of a circle. This is represented by π .

$$\frac{\text{Circumference}}{\text{Diameter}} = \text{constant point} = \pi$$

$$\begin{aligned} \text{Circumference} &= \pi \times \text{dia} \\ &= 2\pi r \text{ (where r is radius of the circle)} \end{aligned}$$

$$\pi = \frac{22}{7}$$

Circumference makes an angle (2πr) = 360°

Radius of the circle makes an angle (r) = 1 Radian

$$\text{ie : } \frac{C}{r} = \frac{360^\circ}{1\text{Radian}}$$

$$\frac{2\pi r}{r} = \frac{360^\circ}{1\text{Radian}}$$

$$2\pi = \frac{360^\circ}{1\text{Radian}}$$

$$2\pi \text{ Radian} = 360^\circ$$

$$\pi \text{ Radian} = 180^\circ$$

$$1 \text{ Radian} = \frac{180^\circ}{\pi}$$

$$1^\circ = \frac{\pi}{180^\circ} \text{ Radian}$$

Examples

1 Convert 45°36'20" into degree and decimal of degree.

$$60 \text{ second} = 1 \text{ minute}$$

$$20 \text{ second} = \frac{20}{60} = 0.333'$$

$$60 \text{ minute} = 1 \text{ degree}$$

$$36.333 \text{ minute} = \frac{36.333}{60} = 0.606^\circ$$

$$45^\circ 36' 20'' = 45.606^\circ$$

2 Convert 24.59° into degree, minute and second

$$1 \text{ degree} = 60 \text{ minute}$$

$$0.59 \text{ degree} = 0.59 \times 60 = 35.4'$$

$$1 \text{ minute} = 60 \text{ second}$$

$$0.4 \text{ minute} = 60 \text{ sec} \times 0.4$$

$$= 24''$$

$$\text{Therefore } 24.59^\circ = 24^\circ 35' 24''$$

3 Change 50°37'30" into degrees

By changing angle degrees into decimals

$$30'' = \frac{30}{60} = 0.50'$$

$$37'30'' = 37.5'$$

$$37.5' = \frac{37.5}{60} = 0.625^\circ$$

$$50^\circ 37' 30'' = 50.625^\circ$$

4 Convert  $23^{\circ}25'32''$  into radians

We know  $1^{\circ} = 60' = 3600''$

Therefore  $23^{\circ}25'32''$

$$= \left( 23 + \frac{25}{60} + \frac{32}{3600} \right) \text{ degrees}$$

$$= \frac{82800 + 1500 + 32}{3600}$$

$$= \frac{84332}{3600}$$

But  $180^{\circ} = \pi$  radians

Therefore  $23.4255$  degrees

$$= \frac{23.4255}{180} \pi \text{ radians}$$

$$= \frac{23.4255}{180} \times \frac{22}{7} \text{ radians}$$

$$= \mathbf{0.4089 \text{ radians}}$$

5 Convert  $87^{\circ}19'57''$  into Radian.

$$19'57'' = 19' + \frac{57''}{60}$$

$$= 19' + 0.95''$$

$$= 19.95'$$

$$87^{\circ}19.95' = 87^{\circ} + \frac{19.95'}{60}$$

$$= 87^{\circ} + 0.332^{\circ} = 87.332^{\circ}$$

$$1^{\circ} = \frac{\pi}{180} \text{ radian}$$

$$87.332^{\circ} = \frac{\pi}{180} \times 87.332 \text{ radian}$$

$$= 1.524 \text{ radian}$$

6 Convert  $67^{\circ}11'43''$  into Radian

$$11'43'' = 11' + \frac{43''}{60}$$

$$= 11' + 0.716''$$

$$= 11.72'$$

$$67^{\circ}11.72' = 67^{\circ} + \frac{11.72'}{60}$$

$$= 67^{\circ} + 0.195^{\circ}$$

$$= 67.2^{\circ}$$

$$1^{\circ} = \frac{\pi}{180} \text{ radian}$$

$$67.2^{\circ} = \frac{\pi}{180} \times 67.2 \text{ radian}$$

$$= 1.173 \text{ radian}$$

7 Convert  $\frac{4}{7} \pi$  radian into degrees

$$1 \text{ radian} = \frac{180}{\pi} \text{ degree}$$

$$\frac{4}{7} \pi \text{ radian} = \frac{180}{\pi} \times \frac{4}{7} \pi \text{ degree}$$

$$= 102.9 \text{ degree}$$

$$= 102^{\circ} 0.9 \times 60'$$

$$= 102^{\circ} 54'$$

8 Convert 0.8357 radian into degrees

$$1 \text{ radian} = \frac{180}{\pi} \text{ degree}$$

$$0.8357 \text{ radian} = \frac{180}{\pi} \times 0.8357 \text{ degree}$$

$$= 47.88^{\circ}$$

$$= 47^{\circ} 0.88 \times 60'$$

$$= 47^{\circ} 52.80'$$

$$= 47^{\circ} 52' 0.8 \times 60''$$

$$= 47^{\circ} 52' 48''$$

9 Convert 2.752 radian into degrees

$$1 \text{ Radian} = \frac{180}{\pi} \text{ degree}$$

$$2.7520 \text{ radian} = \frac{180}{\pi} \times 2.752 \text{ degree}$$

$$= 157.7^{\circ}$$

$$= 157.7^{\circ} \times 60'$$

$$= 157^{\circ} 42'$$

10 Convert  $\frac{3}{5} \pi$  radian into degrees

$$1 \text{ Radian} = \frac{180}{\pi} \text{ degree}$$

$$\frac{3}{5} \pi \text{ radian} = \frac{180}{\pi} \times \frac{3}{5} \pi \text{ degree}$$

$$= 108^{\circ}$$

### Assignment

#### Convert into Degree

1 12 Radian

#### Convert into Radians

2  $78^{\circ}$

3  $47^{\circ}20'$

4  $52^{\circ}36'45''$

5  $25^{\circ}38''$

#### Convert into degree, minute and seconds

6  $46.723^{\circ}$

7  $68.625^{\circ}$

8 0.1269 Radians

9 2.625 Radians

10  $3/5$  Radians

Trigonometry - Trigonometrical ratios

Dependency

The sides of a triangle bear constant ratios for a given definite value of the angle. That is, increase or decrease in the length of the sides will not affect the ratio between them unless the angle is changed. These ratios are trigonometrical ratios. For the given values of the angle a value of the ratios

$\frac{BC}{AB}, \frac{AC}{AB}, \frac{BC}{AC}, \frac{AB}{BC}, \frac{AB}{AC}$  and  $\frac{AC}{BC}$  do not change even when the sides AB, BC, AC are increased to AB', BC' and AC' or decreased to AB'', BC'' and AC''.

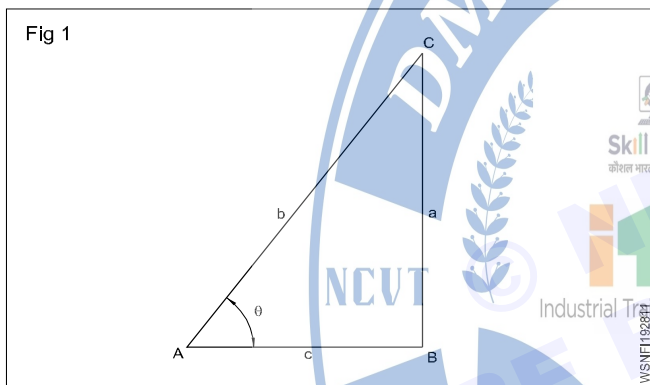
For the angle

AC is the hypotenuse

AB is the adjacent side

BC is the opposite side.

The ratios



The six ratios between the sides have precise definitions.

$$\text{Sine } \theta = \frac{BC}{AC} = \frac{\text{Opposite side}}{\text{Hypotenuse}} = \text{Sin } \theta$$

$$\text{Cosine } \theta = \frac{AB}{AC} = \frac{\text{Adjacent side}}{\text{Hypotenuse}} = \text{Cos } \theta$$

$$\text{Tangent } \theta = \frac{BC}{AB} = \frac{\text{Opposite side}}{\text{Adjacent side}} = \text{Tan } \theta$$

$$\text{Cosecant } \theta = \frac{AC}{BC} = \frac{\text{Hypotenuse}}{\text{Opposite side}} = \text{Cosec } \theta$$

$$\text{Secant } \theta = \frac{AC}{AB} = \frac{\text{Hypotenuse}}{\text{Adjacent side}} = \text{Sec } \theta$$

$$\text{Cotangent } \theta = \frac{AB}{BC} = \frac{\text{Adjacent side}}{\text{Opposite side}} = \text{Cot } \theta$$

Relationship between the ratios

$$\text{Cosec } \theta = \frac{AC}{BC} = \frac{1}{\frac{BC}{AC}} = \frac{1}{\text{sin } \theta}$$

$$\text{sec } \theta = \frac{AC}{AB} = \frac{1}{\frac{AB}{AC}} = \frac{1}{\text{cos } \theta}$$

$$\text{cot } \theta = \frac{AB}{BC} = \frac{1}{\frac{BC}{AB}} = \frac{1}{\text{tan } \theta}$$

$$\text{sin } \theta = \frac{\text{side BC}}{\text{side AC}} = \frac{a}{b}$$

$$\text{cos } \theta = \frac{\text{side AB}}{\text{side AC}} = \frac{c}{b}$$

$$\frac{\text{sin } \theta}{\text{cos } \theta} = \frac{\frac{a}{b}}{\frac{c}{b}} = \frac{a}{b} \times \frac{b}{c} = \frac{a}{c}$$

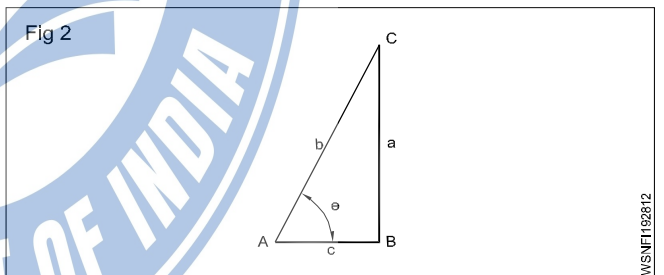
$$= \frac{\text{side BC}}{\text{side AB}} = \text{tan } \theta$$

$$\text{sin } \theta = \frac{1}{\text{cosec } \theta} \text{ or } \text{cosec } \theta = \frac{1}{\text{sin } \theta} \text{ or } \text{sin } \theta \cdot \text{cosec } \theta = 1$$

$$\text{cos } \theta = \frac{1}{\text{sec } \theta} \text{ or } \text{sec } \theta = \frac{1}{\text{cos } \theta} \text{ or } \text{cos } \theta \cdot \text{sec } \theta = 1$$

$$\text{tan } \theta = \frac{1}{\text{cot } \theta} \text{ or } \text{cot } \theta = \frac{1}{\text{tan } \theta} \text{ or } \text{cot } \theta \cdot \text{tan } \theta = 1$$

By pythagoras theorem we have,  $AC^2 = AB^2 + BC^2$



Dividing both sides of the equation by  $AC^2$ , we have

$$\frac{AC^2}{AC^2} = \frac{AB^2}{AC^2} + \frac{BC^2}{AC^2}$$

$$= \left[ \frac{AB}{AC} \right]^2 + \left[ \frac{BC}{AC} \right]^2$$

$$1 = (\text{cos } \theta)^2 + (\text{sin } \theta)^2$$

$$\text{sin}^2 \theta + \text{cos}^2 \theta = 1$$

**Sine, Cosine, Tangent, Cosec, Sec and Cotangent are the six trigonometrical ratios**

$$\text{tan } \theta = \frac{\text{Sin } \theta}{\text{Cos } \theta} \text{ and } \text{sin}^2 \theta + \text{cos}^2 \theta = 1$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

It can be transformed as

$$\sin^2 \theta = 1 - \cos^2 \theta$$

$$\sin \theta = \sqrt{1 - \cos^2 \theta}$$

$$\text{or } \cos^2 \theta = 1 - \sin^2 \theta$$

$$\cos \theta = \sqrt{1 - \sin^2 \theta}$$

$$\tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\sqrt{1 - \cos^2 \theta}}{\cos \theta}$$

$$\tan \theta = \frac{\sin \theta}{\sqrt{1 - \sin^2 \theta}}$$

We know  $\sin^2 \theta + \cos^2 \theta = 1$

Dividing both sides by  $\cos^2 \theta$ .

$$\frac{\sin^2 \theta}{\cos^2 \theta} + \frac{\cos^2 \theta}{\cos^2 \theta} = \frac{1}{\cos^2 \theta}$$

$$\text{or } 1 + \tan^2 \theta = \sec^2 \theta$$

Using the same equation

$$\sin^2 \theta + \cos^2 \theta = 1.$$

Dividing both sides by  $\sin^2 \theta$ ,

$$1 + \frac{\cos^2 \theta}{\sin^2 \theta} = \frac{1}{\sin^2 \theta}$$

$$1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$$

$$1 + \tan^2 \theta = \sec^2 \theta$$

### Trigonometrical Tables

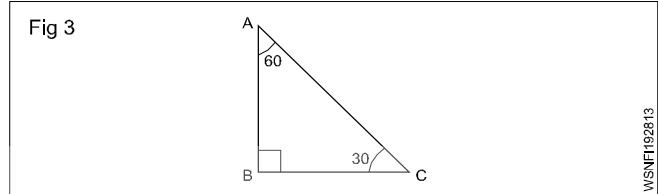
Ratio	0°	30°	45°	60°	90°
sin θ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
tan θ	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

**Cosine of an angle = Sine of its complementary angle**

### Examples

If  $\sin 30^\circ = \frac{1}{2}$  find the value of  $\sin 60^\circ$

By applying pythagores theorem



$$BC^2 = AC^2 - AB^2$$

$$BC^2 = 2^2 - 1^2$$

$$= 4 - 1$$

$$= 3$$

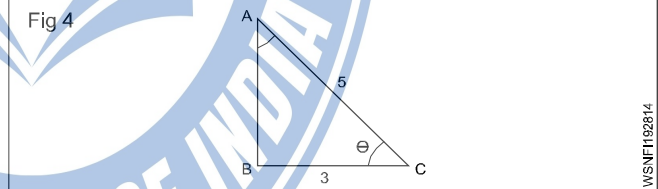
$$BC = \sqrt{3}$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

$\cos \theta = \frac{3}{5}$  Find the other trigonometrical ratios

By applying pythagores theorem

$$\begin{aligned} AB^2 &= AC^2 - BC^2 \\ &= 5^2 - 3^2 = 25 - 9 \\ &= 16 \end{aligned}$$



$$AB = \sqrt{16} = 4$$

$$\text{Now } \sin \theta = \frac{4}{5}$$

$$\tan \theta = \frac{4}{3}$$

$$\operatorname{Cosec} \theta = \frac{5}{4}$$

$$\sec \theta = \frac{5}{3}$$

$$\cot \theta = \frac{3}{4}$$

**When θ increases,**

**Sine value increases;**

**Cosine value decreases;**

**Tangent value increases to more than 1 when the angle is more than 45° (tan60° = 1.732)**

**Sine of an angle = Cosine of its complementary angle**

### Signs of trigonometrical functions for angles more than 90°

Ratio	90 - θ	90 + θ	180 - θ	180 + θ	270 - θ	270 + θ	360 - θ	- θ
sin	cos	cos	sin	- sin	- cos	- cos	- sin	- sin
cos	sin	- sin	- cos	- cos	- sin	sin	cos	cos
tan	cot	- cot	- tan	tan	cot	- cot	- tan	- tan
cosec	sec	sec	cosec	- cosec	- sec	- sec	- cosec	- cosec
sec	cosec	- cosec	- sec	- sec	- cosec	cosec	sec	sec
cot	tan	- tan	- cot	cot	tan	- tan	- cot	- cot

**Simplify :**

$$\begin{aligned} \cot \theta + \tan (180^\circ + \theta) + \tan(90^\circ - \theta) + (\tan 360^\circ - \theta) \\ = \cot \theta + \tan \theta - \cot \theta - \tan \theta \\ = 0 \end{aligned}$$

$$\sec (360^\circ - \theta) = \sec \theta$$

$$\sin (180^\circ + \theta) = - \sin \theta$$

$$\cot (90^\circ + \theta) = - \tan \theta$$

**Simplify :**

$$\frac{\cos (90^\circ + \theta) \sec (-\theta) \tan (180^\circ - \theta)}{\sec (360^\circ - \theta) \sin (180^\circ + \theta) \cos (90^\circ - \theta)}$$

$$= \frac{(-\sin \theta) \times (\sec \theta) \times (-\tan \theta)}{(\sec \theta) \times (-\sin \theta) \times (-\sin \theta)}$$

$$= \frac{\tan \theta}{\sin \theta} = \frac{1}{\cos \theta} = \sec \theta$$

$$\frac{\cos (90^\circ + \theta) \sec (-\theta) \tan (180^\circ - \theta)}{\sec (360^\circ - \theta) \sin (180^\circ + \theta) \cos (90^\circ - \theta)}$$

$$= \frac{(-\sin \theta)(\sec \theta)(\tan \theta)}{(\sec \theta)(-\sin \theta)(-\tan \theta)}$$

$$= 1$$

**simplify:**

$$\frac{\cos (90^\circ + \theta) \sec (-\theta) \tan (180^\circ - \theta)}{\sec (360^\circ - \theta) \sin (180^\circ + \theta) \cos (90^\circ - \theta)}$$

$$\cos (90^\circ + \theta) = - \sin \theta$$

$$\sec (-\theta) = \sec \theta$$

$$\tan (180^\circ - \theta) = - \tan \theta$$

**Simplify:**

$$\cot \theta + \tan (180^\circ + \theta) + \tan (90^\circ + \theta) + \tan (360^\circ - \theta)$$

$$\tan (180^\circ - \theta) = \tan \theta$$

$$\tan (90^\circ + \theta) = - \cot \theta$$

$$\tan (360^\circ - \theta) = - \tan \theta$$

$$\cot \theta + \tan (180^\circ + \theta) + \tan (90^\circ + \theta) + \tan (360^\circ - \theta)$$

$$\cot \theta + \tan \theta - \cot \theta - \tan \theta = 0$$

### Assignment

1 Given  $\sin 30^\circ = 1/2$ , find the value of  $\tan 60^\circ$

2 If  $\cos \theta = 4/5$ , find the other ratios

3 If  $\sin A = 3/5$ , find  $\cos \theta$ ,  $\tan \theta$  &  $\sec \theta$

4 If  $\tan \theta = 24/7$ , find  $\sin \theta$  and  $\cos \theta$

5 Find the value of  $\cos \theta$  and  $\tan \theta$ , if  $\sin \theta = 1/2$

6 If  $\cos \theta = 5/13$ , find the value of  $\tan \theta$

7 If  $\sin \theta = 1/2$ , find the value of  $\sin^2 \theta - \cos^2 \theta$

8 What is the value of

$$\frac{\sin^2 30^\circ}{\cos^2 45^\circ} + \frac{\tan 45^\circ}{\sec 60^\circ} - \frac{\sin 60^\circ}{\cot 45^\circ} - \frac{\cos 30^\circ}{\sin 90^\circ}$$

**Simplify :**

1  $\tan (90^\circ + A) + (\tan 180^\circ + A) \tan (90^\circ + A)$

2  $\frac{\cos (90^\circ + \theta) \cdot \sec (-\theta) \cdot \tan (180^\circ - \theta)}{\sec (360^\circ + \theta) \cdot \sin (180^\circ + \theta) \cdot \cot (90^\circ + \theta)}$

Trigonometry - Trigonometrical tables

Use of trigonometrical tables (Ref: Sin, Cos & Tan Table)

Deg.	Minutes from 0 to 4						Mean difference				
	0'	6'	12'	18'	24'	54'	1'	2'	3'	4'	5'
0				.			.				
1				.			.				
2				.			.				
3				.			.				
.				.			.				
.				.			.				
.				.			.				
26	..	..	..	x	..	..	..	5			
.											
.											
.											
89											

$$\begin{aligned} \cos 43^\circ - 41' &= \text{value for } \cos 43^\circ .36' \\ &\quad - \text{the value given for} \\ \text{mean difference of } 5' &= 0.7242 - 0.0010 \\ &= 0.7232 \end{aligned}$$

**When reading sine value add the mean difference value. When reading cosine value subtract the mean difference value.**

Arrangement

Values of trigonometrical ratios can be taken from mathematical tables.

The left hand vertical column consists of degrees.

The top horizontal column is arranged in minutes in steps of 6' from 0' to 54'. In the extreme right horizontal columns the mean differences are written in minutes from 1' to 5' in steps of 1' to account for angles with minutes between the interval of 6'.

- The values of cosine, cosecant and cotangent decrease when the value of the angle increases.
- For sine, secant and tangent, the value increases when the angle increases.
- The value of sine and cosine will never be more than 1.
- The value of secant and cosecant will never be less than 1.
- The value of Tan and Cot ranges from 0 to ∞.

Sine value for 26° – 20'

Refer to Natural sine table.

Degrees column go up to 26° down

Minutes column 18' horizontal and under this note the value which is given as 0.4431.

Under mean difference for 2' in the same horizontal line 5 is given. Add this to the extreme right number noted for 26° – 18'.

$$\text{Sine } 26^\circ - 20' = 0.4431 + .0005 = 0.4436$$

Cosine value for 43° – 41'

Referring to the Natural cosines table for 43° – 36' it is given as 0.7242 and the mean difference for 5' minutes is given as 10.

EXAMPLE

From the tables obtain the cosine of 45° – 20'.

$$\cos 45^\circ - 18' = 0.7108$$

$$\text{mean difference for } 2' = 0.0004$$

$$\cos 45^\circ - 20' = 0.7104$$

SINE TABLE

1 Sin 25° = 0.4226

23 Sin 17° 5'

sin 17°	=	0.2924
Difference 5'	=	14
sin 17° 5'	=	0.2938

Ans

3 sin 17° 45' 13"

$$\sin 17^\circ 46' = 0.3051$$

$$\sin 17^\circ 45' = 0.3048$$

$$\text{Difference } 1' = 0.0003$$

$$\begin{aligned}
 1'(\text{or}) 60'' &= 0.0003 \\
 13'' &= \frac{0.0003}{60} \times 13 \\
 &= \frac{0.0039}{60} \\
 &= \frac{0.00039}{6} \\
 &= 0.000065
 \end{aligned}$$

$$\begin{aligned}
 \sin 17^\circ 45' &= 0.3048 \\
 13'' &= 0.000065
 \end{aligned}$$

$$\sin 17^\circ 45' 13'' = 0.304865 \quad \text{Ans.}$$

#### 4 $\sin 82^\circ 14' 18''$

$$\begin{aligned}
 \sin 82^\circ 15' &= 0.9908 \\
 \sin 82^\circ 14' &= 0.9908 \\
 \text{Difference } 1' &= 0
 \end{aligned}$$

$$\begin{aligned}
 1'(\text{or}) 60'' &= 0 \\
 \sin 18'' &= 0
 \end{aligned}$$

$$\begin{aligned}
 \sin 82^\circ 14' &= 0.9908 \\
 18'' &= 0.0000
 \end{aligned}$$

$$\sin 82^\circ 41' 18'' = 0.9908 \quad \text{Ans.}$$

**Finding the corresponding angles when sine values are given:**

1.  $\sin \theta = 0.9925$

$$\theta = 83^\circ$$

2.  $\sin \theta = 0.8791$

$$0.8788 = \sin 61^\circ 30'$$

$$0.0003 = 2'$$

$$0.8791 = \sin 61^\circ 32'$$

3.  $\sin \theta = 0.68015$

$$0.6794 = \sin 42^\circ 48'$$

$$0.0006 = 3'$$

$$0.6800 = \sin 42^\circ 51'$$

$$0.6803 = \sin 42^\circ 52'$$

$$\text{Difference } 0.0003 = 1'(\text{or}) 60''$$

$$\begin{aligned}
 0.00015 &= \frac{60}{0.0003} \times 0.00015 \\
 &= \frac{60 \times 15}{30} \\
 &= 30''
 \end{aligned}$$

$$0.6800 = \sin 42^\circ 51'$$

$$0.00015 = 30''$$

$$0.68015 = \sin 42^\circ 51' 30''$$

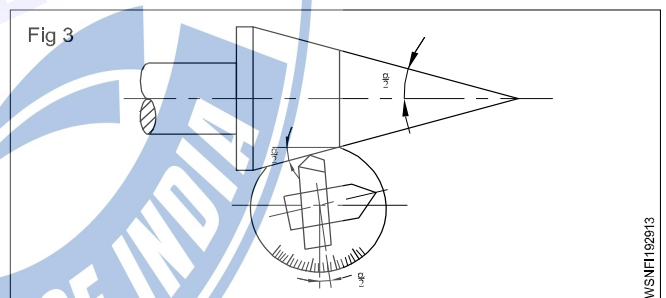
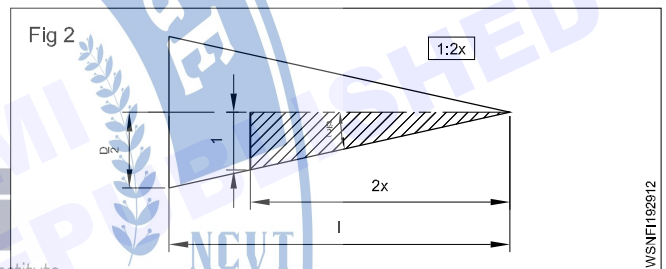
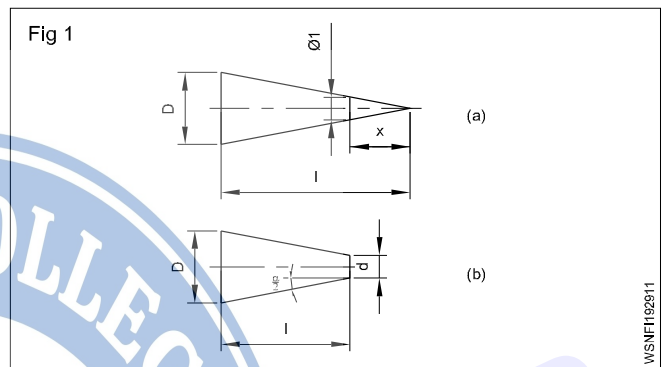
$$\theta = 42^\circ 51' 30''$$

### Calculations involving tapers

**D** - Big diameter of the taper

**d** - small diameter of the taper

**C** - Taper Ratio - 1:x



$\frac{C}{2}$  Ratio of inclination - 1:2 x

**l** - length of taper

**a** - included angle of taper

$\frac{\alpha}{2}$  - setting angle

Taper ratio = Ratio of inclination (for wedges).

### Taper ratio

The ratio between the difference in diameter to the length of the taper is known as taper ratio. **D** is the difference in larger diameter shown in the sketch as the small diameter of taper is 0. Taper ratio is **D** : **l**. In the sectioned portion the difference in diameter is 1 and the length of taper is shown as **x**.

$C = D : d = 1 : x$  as per Fig 1 (a),  $C = \frac{D-d}{l}$  as per Fig 1 (b)

### Ratio of inclination

Taking half of the taper,  $\frac{D}{2}$  is the difference in diameter for a taper length of  $l$ , if  $d = 0$ .

$$\therefore \frac{C}{2} = \frac{D}{2l} \quad \text{if the small diameter is 0}$$

$$\text{or } \frac{C}{2} = \frac{D-d}{2l}$$

1 Ratio of inclination =  $\frac{1}{2}$  of the taper ratio.

### Setting angle

One of the methods of turning taper is by swivelling the compound slide to an angle known as setting angle and feeding the tool at an angle to the axis of work.

$$\tan \frac{\alpha}{2} = \frac{C}{2} = \frac{D-d}{2l}$$

$$\tan \frac{\alpha}{2} = \frac{\text{taper ratio}}{2}$$

$$= \frac{\text{difference in diameter}}{2 \times \text{taper length}}$$

NOTE:  $\frac{\alpha}{2}$  is the setting angle which is equal to half of the included angle of the taper.

### Cos Table

1	<b>Cos 38°</b>	
	Cos 38°	= 0.7880
2	<b>Cos 83°12'</b>	
	Cos 83°12'	= 0.1184
3	<b>Cos 26°40'</b>	
	Cos 26°36'	= 0.8942
	4' =	5(-)
	<b>Cos 26°40'</b>	= 0.8937
4	<b>Cos 31°20'</b>	
	Cos 31°18'	= 0.8545
	2' =	3(-)
	<b>Cos 31°20'</b>	= 0.8542

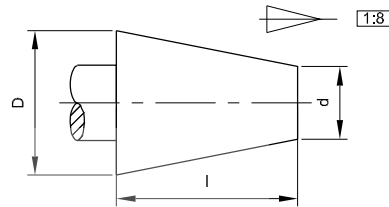
Taper Ratio  $C = 1 : x$  or  $D : l$  or  $(D-d) : l$

Ratio of inclination  $\frac{C}{2} = 1 : 2x = \frac{D}{2} : l$  or  $\left(\frac{D-d}{2}\right) : l$

Setting angle determination is by the formula

$$\tan \frac{\alpha}{2} = \frac{D-d}{2l} = \frac{C}{2}$$

Fig 4



### EXAMPLE

A pivot in the form of a frustum of a cone has a taper ratio 1:8. If the small diameter is 30 mm and length of taper is 80 mm, find its large diameter.

$$C = 1:8 = \frac{1}{8}$$

$$\frac{D-d}{l} = \frac{1}{8}$$

$$\therefore D-d = \frac{1}{8} \times 80 = 10 \text{ mm.}$$

$$D - 30 \text{ mm} = 10 \text{ mm}$$

$$D = 10 \text{ mm} + 30 \text{ mm} = 40 \text{ mm}$$

$$\text{Large diameter } D = 40 \text{ mm}$$

Find the corresponding angles when cos values are given:

$$1 \quad \text{Cos } \theta = 0.5150$$

$$\theta = 59^\circ$$

$$2 \quad \text{Cos } \theta = 0.0192$$

$$\theta = 88^\circ 54'$$

$$3 \quad \text{Cos } \theta = 0.9682$$

$$0.9686 = \cos 14^\circ 24'$$

$$(-) \quad 4 = 5'$$

$$0.9682 = \cos 14^\circ 29'$$

$$\theta = 14^\circ 29'$$

4  $\cos \theta = 0.8476$

$0.8480 = \cos 32^\circ 0'$

(-)  $0.0003 = 2'$

$0.8477 = \cos 32^\circ 2'$   
 $0.8475 = \cos 32^\circ 3'$

$0.0002 = 1' \text{ (or) } 60''$   
 $60$

$0.0001 = \frac{\text{-----}}{0.0002} \times 0.0001$   
 $60$

$= \text{-----} \times 1$   
 $2$

$= 30''$

$0.8477 = \cos 32^\circ 2'$

(-)  $0.0001 = 30'' \text{ (+)}$

**0.8476 = Cos 32° 2' 30"**

**tan Table**

**1 tan 35° 37'**

$\tan 35^\circ 36' = 0.7159$

$1' = 0.0004$

$\tan 35^\circ 37' = 0.7163$

**2 tan 50° 5'**

$\tan 50^\circ 0' = 1.1918$

$5' = 0.0036$

$\tan 50^\circ 5' = 1.1954$

**Find the corresponding angles when tan values are given**

1  $\tan \theta = 0.3972$

$0.3959 = \tan 21^\circ 36'$

$0.0013 = 4'$

$0.3972 = \tan 21^\circ 40'$

2  $\tan \theta = 1.0065$

$1.0035 = \tan 45^\circ 6'$

$0.0030 = 5'$

$1.0065 = \tan 45^\circ 11'$

**Problems Related with Trigonometrical tables**

1 A 250 mm Sine bar is used to measure an angle. If the difference in height is 5 cm, find the angle.

$$\sin \theta = \frac{\text{Opp.side}}{\text{Hyp.}} = \frac{h}{l}$$

$= \frac{5 \text{ cm}}{250 \text{ mm}}$

$= \frac{50 \text{ mm}}{250 \text{ mm}}$

$= 0.2000$

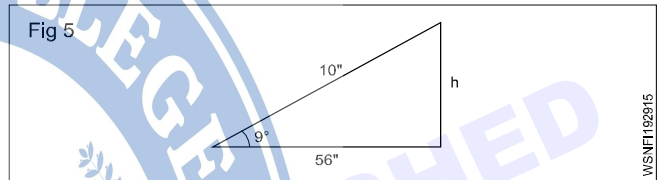
$\theta = 11^\circ 32'$

2 Find the height of the slip gauge if a Sine bar with plugs of 10" centre is set up to inspect a taper having an included angle of 9° 56".

$$\sin \theta = \frac{\text{Opp.side}}{\text{Hyp.}} = \frac{h}{l}$$

$\sin 9^\circ 56'' = \frac{h}{10}$

$\therefore h = 10 \times \sin 9^\circ 56''$



$\sin 9^\circ 0'' = 0.1564$

$\sin 9^\circ 1'' = 0.1567$

$1' \text{ (or) } 60'' = 0.0003$

$56'' = \frac{0.0003}{60} \times 56$

$= \frac{0.0168}{60}$

$= \frac{0.00168}{6}$

$= 0.00028$

$\sin 9^\circ 0' 00'' = 0.1564$

$56'' = 0.00028$

$\sin 9^\circ 0' 56'' = 0.15668$

$h = 10 \times \sin 9^\circ 56''$

$= 10 \times 0.15668$

$= 1.5668 \text{ cm}$

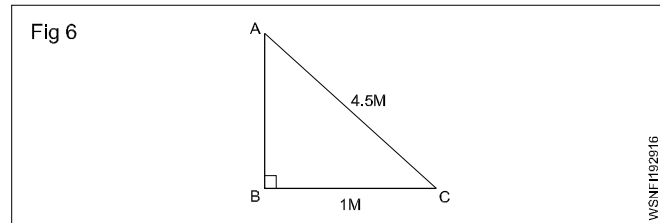
**Height of slip gauge = 1.5668"**

3 Find the angle which the ladder makes with the ground if the foot of a 4.5 m long ladder is placed at 1 m away from the wall.

In Right angled D

$$\cos C = \frac{BC}{AC}$$

$$\begin{aligned} \cos \theta &= \frac{1 \text{ m}}{4.5 \text{ m}} = 0.2222 \\ \cos &= 0.2222 \\ 0.2233 &= \cos 77^\circ 6' \\ (-) \quad 0.0011 &= \quad \quad \quad 4' (+) \\ \hline 0.2222 &= \cos 77^\circ 10' \\ \hline \theta &= 77^\circ 10' \end{aligned}$$



## Assignment

### I Find the values of the given angles

- 1 Sin 65°
- 2 Sin 42°23'
- 3 Sin 66°35'32"
- 4 Sin 7°15'41"
- 5 Sin 27°27'
- 6 Cos 47°39'
- 7 Cos 47°39'
- 8 Cos 79°31'53"
- 9 Tan 28°45'
- 10 Tan 67°27'36"

### II Find corresponding angles for given values

- 1 Sin  $\theta$  = 0.3062
- 2 Sin  $\theta$  = 0.6002
- 3 Sin  $\theta$  = 0.22453
- 4 Sin  $\theta$  = 0.04802
- 5 Cos  $\theta$  = 0.6446

6 Cos  $\theta$  = 0.8926

7 Cos  $\theta$  = 0.11773

8 Cos  $\theta$  = 0.21646

9 Tan  $\theta$  = 0.3411

10 Tan  $\theta$  = 2.3868

### III

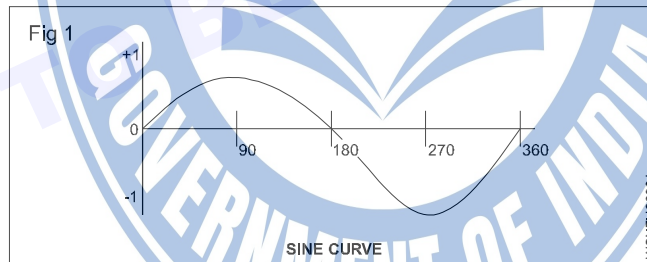
- 1 Calculate its base, if the slant height of a cone is 12.25 cm and the vertex angle is 110°.
- 2 A ladder 2.5 m long makes an angle of 60° with the ground. Find the height of the wall where the ladder touches the wall.
- 3 A sine bar of 200 mm is to be set at an angle of 15°15'3". Select the slip gauge block to built up the required height.
- 4 In a right angled triangle ABC,  $\angle C = 90^\circ$ , If AB = 50 mm and  $\angle B = 75^\circ$ , Find the remaining sides.
- 5 Calculate the required length of the bar for this point if a centre point having an included angle of 60° is to be turned at the end of a 50 mm dia bar.

**Natural Sines**

°	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'	1'	2'	3'	4'	5'
	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°					
0	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157	3	6	9	12	15
1	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332	3	6	9	12	15
2	0.0349	0.0366	0.0384	0.0401	0.0419	0.0436	0.0454	0.0471	0.0488	0.0506	3	6	9	12	15
3	0.0523	0.0541	0.0558	0.0576	0.0593	0.0610	0.0628	0.0645	0.0663	0.0680	3	6	9	12	15
4	0.0698	0.0715	0.0732	0.0750	0.0767	0.0785	0.0802	0.0819	0.0837	0.0854	3	6	9	12	14
5	0.0872	0.0899	0.0906	0.0924	0.0941	0.0958	0.0976	0.0993	0.1011	0.1028	3	6	9	12	14
6	0.1045	0.1063	0.1080	0.1097	0.1115	0.1132	0.1149	0.1167	0.1184	0.1201	3	6	9	12	14
7	0.1219	0.1236	0.1253	0.1271	0.1288	0.1305	0.1323	0.1340	0.1357	0.1374	3	6	9	12	14
8	0.1392	0.1409	0.1426	0.1444	0.1461	0.1478	0.1495	0.1513	0.1530	0.1547	3	6	9	11	14
9	0.1564	0.1582	0.1599	0.1616	0.1633	0.1650	0.1668	0.1685	0.1702	0.1719	3	6	9	11	14
10	0.1736	0.1754	0.1771	0.1788	0.1805	0.1822	0.1840	0.1857	0.1874	0.1891	3	6	9	11	14
11	0.1908	0.1925	0.1942	0.1959	0.1977	0.1994	0.2011	0.2028	0.2045	0.2062	3	6	9	12	14
12	0.2079	0.2096	0.2113	0.2130	0.2147	0.2164	0.2181	0.2198	0.2215	0.2232	3	6	9	11	14
13	0.2250	0.2267	0.2284	0.2300	0.2317	0.2334	0.2351	0.2368	0.2385	0.2402	3	6	8	11	14
14	0.2419	0.2436	0.2453	0.2470	0.2487	0.2504	0.2521	0.2538	0.2554	0.2571	3	6	8	11	14
15	0.2558	0.2605	0.2622	0.2639	0.2656	0.2672	0.2689	0.2706	0.2723	0.2740	3	6	8	11	14
16	0.2756	0.2773	0.2790	0.2807	0.2823	0.2840	0.2857	0.2874	0.2890	0.2907	3	6	8	11	14
17	0.2924	0.2940	0.2957	0.2974	0.2990	0.3007	0.3024	0.3040	0.3057	0.3074	3	6	8	11	14
18	0.3090	0.3107	0.3123	0.3140	0.3156	0.3173	0.3190	0.3206	0.3223	0.3239	3	6	8	11	14
19	0.3256	0.3272	0.3289	0.3305	0.3322	0.3338	0.3355	0.3371	0.3387	0.3404	3	5	8	11	14
20	0.3420	0.3437	0.3453	0.3469	0.3486	0.3502	0.3518	0.3535	0.3551	0.3567	3	5	8	11	14
21	0.3584	0.3600	0.3616	0.3633	0.3649	0.3665	0.3681	0.3697	0.3714	0.3730	3	5	8	11	14
22	0.3746	0.3762	0.3778	0.3795	0.3811	0.3827	0.3843	0.3859	0.3875	0.3891	3	5	8	11	13
23	0.3907	0.3923	0.3939	0.3955	0.3971	0.3987	0.4003	0.4019	0.4035	0.4051	3	5	8	11	13
24	0.4067	0.4083	0.4099	0.4115	0.4131	0.4147	0.4163	0.4179	0.4195	0.4210	3	5	8	11	13
25	0.4226	0.4242	0.4258	0.4274	0.4289	0.4305	0.4321	0.4337	0.4352	0.4368	3	5	8	11	13
26	0.4384	0.4399	0.4415	0.4431	0.4446	0.4462	0.4478	0.4493	0.4509	0.4524	3	5	8	10	13
27	0.4540	0.4555	0.4571	0.4586	0.4602	0.4617	0.4633	0.4648	0.4664	0.4679	3	5	8	10	13
28	0.4695	0.4710	0.4726	0.4741	0.4756	0.4772	0.4787	0.4802	0.4818	0.4833	3	5	8	10	13
29	0.4848	0.4863	0.4879	0.4894	0.4909	0.4924	0.4939	0.4955	0.4970	0.4985	3	5	8	10	13
30	0.5000	0.5015	0.5030	0.5045	0.5060	0.5075	0.5090	0.5105	0.5120	0.5135	3	5	8	10	13
31	0.5150	0.5165	0.5180	0.5195	0.5210	0.5225	0.5240	0.5255	0.5270	0.5284	2	5	7	10	12
32	0.5299	0.5314	0.5329	0.5344	0.5358	0.5373	0.5388	0.5402	0.5417	0.5432	2	5	7	10	12
33	0.5446	0.5461	0.5476	0.5490	0.5505	0.5519	0.5534	0.5548	0.5563	0.5577	2	5	7	10	12
34	0.5592	0.5606	0.5621	0.5635	0.5650	0.5664	0.5678	0.5693	0.5707	0.5721	2	5	7	10	12
35	0.5736	0.5750	0.5764	0.5779	0.5793	0.5807	0.5821	0.5835	0.5850	0.5864	2	5	7	9	12
36	0.5878	0.5892	0.5906	0.5920	0.5934	0.5948	0.5962	0.5976	0.5990	0.6004	2	5	7	9	12
37	0.6018	0.6032	0.6046	0.6060	0.6074	0.6088	0.6101	0.6115	0.6129	0.6143	2	5	7	9	12
38	0.6157	0.6170	0.6184	0.6198	0.6211	0.6225	0.6239	0.6252	0.6266	0.6280	2	5	7	9	11
39	0.6293	0.6307	0.6320	0.6334	0.6347	0.6361	0.6374	0.6388	0.6401	0.6414	2	4	7	9	11
40	0.6428	0.6441	0.6455	0.6468	0.6481	0.6494	0.6508	0.6521	0.6534	0.6547	2	4	7	9	11
41	0.6561	0.6574	0.6587	0.6600	0.6613	0.6626	0.6639	0.6652	0.6665	0.6678	2	4	7	9	11
42	0.6691	0.6704	0.6717	0.6730	0.6743	0.6756	0.6769	0.6782	0.6794	0.6807	2	4	6	9	11
43	0.6820	0.6833	0.6845	0.6858	0.6871	0.6884	0.6896	0.6909	0.6921	0.6934	2	4	6	8	11
44	0.6947	0.6959	0.6972	0.6984	0.6997	0.7009	0.7022	0.7034	0.7046	0.7059	2	4	6	8	10
45	0.7071	0.7083	0.7096	0.7108	0.7120	0.7133	0.7145	0.7157	0.7169	0.7181	2	4	6	8	10
46	0.7193	0.7206	0.7218	0.7230	0.7242	0.7254	0.7266	0.7278	0.7290	0.7302	2	4	6	8	10
47	0.7314	0.7325	0.7337	0.7349	0.7361	0.7373	0.7385	0.7396	0.7408	0.7420	2	4	6	8	10
48	0.7431	0.7443	0.7455	0.7466	0.7478	0.7490	0.7501	0.7513	0.7524	0.7536	2	4	6	8	10
49	0.7547	0.7558	0.7570	0.7581	0.7593	0.7604	0.7615	0.7627	0.7638	0.7649	2	4	6	8	9
50	0.7660	0.7672	0.7683	0.7694	0.7705	0.7716	0.7727	0.7738	0.7749	0.7760	2	4	6	7	9
51	0.7771	0.7782	0.7793	0.7804	0.7815	0.7826	0.7837	0.7848	0.7859	0.7869	2	4	5	7	9
52	0.7880	0.7891	0.7902	0.7912	0.7923	0.7934	0.7944	0.7955	0.7965	0.7976	2	4	5	7	9
53	0.7986	0.7997	0.8007	0.8018	0.8028	0.8039	0.8049	0.8059	0.8070	0.8080	2	3	5	7	9
54	0.8090	0.8100	0.8111	0.8121	0.8131	0.8141	0.8151	0.8161	0.8171	0.8181	2	3	5	7	8
55	0.8192	0.8202	0.8211	0.8221	0.8231	0.8241	0.8251	0.8261	0.8271	0.8281	2	3	5	7	8
56	0.8290	0.8300	0.8310	0.8320	0.8329	0.8339	0.8348	0.8358	0.8368	0.8377	2	3	5	6	8
57	0.8387	0.8396	0.8406	0.8415	0.8425	0.8434	0.8443	0.8453	0.8462	0.8471	2	3	5	6	8
58	0.8480	0.8490	0.8499	0.8508	0.8517	0.8526	0.8536	0.8545	0.8554	0.8563	2	3	5	6	8
59	0.8572	0.8581	0.8590	0.8599	0.8607	0.8616	0.8625	0.8634	0.8643	0.8652	1	3	4	6	7

## Natural Sines

°	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'	1'	2'	3'	4'	5'
	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°					
60	0.8660	0.8669	0.8678	0.8686	0.8695	0.8704	0.8712	0.8721	0.8729	0.8738	1	3	4	6	7
61	0.8746	0.8755	0.8763	0.8771	0.8780	0.8788	0.8796	0.8805	0.8813	0.8821	1	3	4	6	7
62	0.8829	0.8838	0.8846	0.8854	0.8862	0.8870	0.8878	0.8886	0.8894	0.8902	1	3	4	5	7
63	0.8910	0.8918	0.8926	0.8934	0.8942	0.8949	0.8957	0.8965	0.8973	0.8980	1	3	4	5	6
64	0.8988	0.8996	0.9003	0.9011	0.9018	0.9026	0.9033	0.9041	0.9048	0.9056	1	3	4	5	6
65	0.9063	0.9070	0.9078	0.9085	0.9092	0.9100	0.9107	0.9114	0.9121	0.9128	1	2	4	5	6
66	0.9135	0.9143	0.9150	0.9157	0.9164	0.9171	0.9178	0.9184	0.9191	0.9198	1	2	3	5	6
67	0.9205	0.9212	0.9219	0.9225	0.9232	0.9239	0.9245	0.9252	0.9259	0.9265	1	2	3	4	6
68	0.9272	0.9278	0.9285	0.9291	0.9298	0.9304	0.9311	0.9317	0.9323	0.9330	1	2	3	4	5
69	0.9336	0.9342	0.9348	0.9354	0.9361	0.9367	0.9373	0.9379	0.9385	0.9391	1	2	3	4	5
70	0.9397	0.9403	0.9409	0.9415	0.9421	0.9426	0.9432	0.9438	0.9444	0.9449	1	2	3	4	5
71	0.9455	0.9461	0.9466	0.9472	0.9478	0.9483	0.9489	0.9494	0.9500	0.9505	1	2	3	4	5
72	0.9511	0.9516	0.9521	0.9527	0.9532	0.9537	0.9542	0.9548	0.9553	0.9558	1	2	3	3	4
73	0.9563	0.9568	0.9573	0.9578	0.9583	0.9588	0.9593	0.9598	0.9603	0.9608	1	2	2	3	4
74	0.9613	0.9617	0.9622	0.9627	0.9632	0.9636	0.9641	0.9646	0.9650	0.9655	1	2	2	3	4
75	0.9659	0.9664	0.9668	0.9673	0.9677	0.9681	0.9686	0.9690	0.9694	0.9699	1	1	2	3	4
76	0.9703	0.9707	0.9711	0.9715	0.9720	0.9724	0.9728	0.9732	0.9736	0.9740	1	1	2	3	3
77	0.9744	0.9748	0.9751	0.9755	0.9759	0.9763	0.9767	0.9770	0.9774	0.9778	1	1	2	2	3
78	0.9781	0.9785	0.9789	0.9792	0.9796	0.9799	0.9803	0.9806	0.9810	0.9813	1	1	2	2	3
79	0.9816	0.9820	0.9823	0.9826	0.9829	0.9833	0.9836	0.9839	0.9842	0.9845	1	1	2	2	3
80	0.9848	0.9851	0.9854	0.9857	0.9860	0.9863	0.9866	0.9869	0.9871	0.9874	0	1	1	2	2
81	0.9877	0.9880	0.9882	0.9885	0.9888	0.9890	0.9893	0.9895	0.9898	0.9900	0	1	1	2	2
82	0.9903	0.9905	0.9907	0.9910	0.9912	0.9914	0.9917	0.9919	0.9921	0.9923	0	1	1	1	2
83	0.9925	0.9928	0.9930	0.9932	0.9934	0.9936	0.9938	0.9940	0.9942	0.9943	0	1	1	1	2
84	0.9945	0.9947	0.9949	0.9951	0.9952	0.9954	0.9956	0.9957	0.9959	0.9960	0	1	1	1	1
85	0.9962	0.9963	0.9965	0.9966	0.9968	0.9969	0.9971	0.9972	0.9973	0.9974	0	0	1	1	1
86	0.9976	0.9977	0.9978	0.9979	0.9980	0.9981	0.9982	0.9983	0.9984	0.9985	0	0	1	1	1
87	0.9986	0.9987	0.9988	0.9989	0.9990	0.9990	0.9991	0.9992	0.9993	0.9993	0	0	0	1	1
88	0.9994	0.9995	0.9995	0.9996	0.9996	0.9997	0.9997	0.9997	0.9998	0.9998	0	0	0	0	0
89	0.9998	0.9999	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	0	0	0	0	0
90	1.0000														



Quadrant	Angle	$\sin A =$	Examples
First	0 to 90°	$\sin A$	$\sin 34^\circ 38' = 0.5683$
Second	90° to 180°	$\sin(180^\circ - A)$	$\sin 145^\circ 22' = \sin(180^\circ - 145^\circ 22')$ $= \sin 34^\circ 38' = 0.5683$
Third	180° to 270°	$-\sin(A - 180^\circ)$	$\sin 214^\circ 38' = -\sin(214^\circ 38' - 180^\circ)$ $= -\sin 34^\circ 38' = -0.5683$
Fourth	270° to 360°	$-\sin(360^\circ - A)$	$\sin 325^\circ 22' = -\sin(360^\circ - 325^\circ 22')$ $= -\sin 34^\circ 38' = -0.5683$

**Natural Cosines**

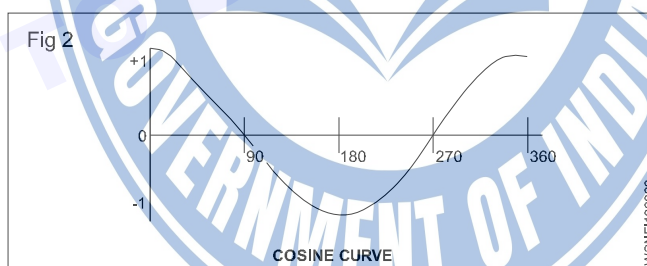
**Numbers in different columns to be subtracted, not added**

°	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'	1'	2'	3'	4'	5'
	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°					
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0.9999	0.9999	0.9999	0.9999	0	0	0	0	0
1	0.9998	0.9998	0.9998	0.9997	0.9997	0.9997	0.9996	0.9996	0.9995	0.9995	0	0	0	0	0
2	0.9994	0.9993	0.9993	0.9992	0.9991	0.9990	0.9990	0.9989	0.9988	0.9987	0	0	0	1	1
3	0.9986	0.9985	0.9984	0.9983	0.9982	0.9981	0.9980	0.9979	0.9978	0.9977	0	0	1	1	1
4	0.9976	0.9974	0.9973	0.9972	0.9971	0.9969	0.9968	0.9966	0.9965	0.9963	0	0	1	1	1
5	0.9962	0.9960	0.9959	0.9957	0.9956	0.9954	0.9952	0.9951	0.9949	0.9947	0	1	1	1	1
6	0.9945	0.9943	0.9942	0.9940	0.9938	0.9936	0.9934	0.9932	0.9930	0.9928	0	1	1	1	2
7	0.9925	0.9923	0.9921	0.9919	0.9917	0.9914	0.9912	0.9910	0.9907	0.9905	0	1	1	1	2
8	0.9903	0.9900	0.9898	0.9895	0.9893	0.9890	0.9888	0.9885	0.9882	0.9880	0	1	1	2	2
9	0.9877	0.9874	0.9871	0.9869	0.9866	0.9863	0.9860	0.9857	0.9854	0.9851	0	1	1	2	2
10	0.9848	0.9845	0.9842	0.9839	0.9836	0.9833	0.9829	0.9826	0.9823	0.9820	1	1	2	2	3
11	0.9816	0.9813	0.9810	0.9806	0.9803	0.9799	0.9796	0.9792	0.9789	0.9785	1	1	2	2	3
12	0.9781	0.9778	0.9774	0.9770	0.9767	0.9763	0.9759	0.9755	0.9751	0.9748	1	1	2	2	3
13	0.9744	0.9740	0.9736	0.9732	0.9728	0.9724	0.9720	0.9715	0.9711	0.9707	1	1	2	3	3
14	0.9703	0.9699	0.9694	0.9690	0.9686	0.9681	0.9677	0.9673	0.9668	0.9664	1	1	2	3	4
15	0.9659	0.9655	0.9650	0.9646	0.9641	0.9636	0.9632	0.9627	0.9622	0.9617	1	2	2	3	4
16	0.9613	0.9608	0.9603	0.9598	0.9593	0.9588	0.9583	0.9578	0.9573	0.9568	1	2	2	3	4
17	0.9563	0.9558	0.9553	0.9548	0.9542	0.9537	0.9532	0.9527	0.9521	0.9516	1	2	3	3	4
18	0.9511	0.9505	0.9500	0.9494	0.9489	0.9483	0.9478	0.9472	0.9466	0.9461	1	2	3	4	5
19	0.9455	0.9449	0.9444	0.9438	0.9432	0.9426	0.9421	0.9415	0.9409	0.9403	1	2	3	4	5
20	0.9397	0.9391	0.9385	0.9379	0.9373	0.9367	0.9361	0.9354	0.9348	0.9342	1	2	3	4	5
21	0.9336	0.9330	0.9323	0.9317	0.9311	0.9304	0.9298	0.9291	0.9285	0.9278	1	2	3	4	5
22	0.9272	0.9265	0.9259	0.9252	0.9245	0.9239	0.9232	0.9225	0.9219	0.9212	1	2	3	4	6
23	0.9205	0.9198	0.9191	0.9184	0.9178	0.9171	0.9164	0.9157	0.9150	0.9143	1	2	3	5	6
24	0.9135	0.9128	0.9121	0.9114	0.9107	0.9100	0.9092	0.9085	0.9078	0.9070	1	2	4	5	6
25	0.9063	0.9056	0.9048	0.9041	0.9033	0.9026	0.9018	0.9011	0.9003	0.8996	1	3	4	5	6
26	0.8988	0.8980	0.8973	0.8965	0.8957	0.8949	0.8942	0.8934	0.8926	0.8918	1	3	4	5	6
27	0.8910	0.8902	0.8894	0.8886	0.8878	0.8870	0.8862	0.8854	0.8846	0.8838	1	3	4	5	7
28	0.8829	0.8821	0.8813	0.8805	0.8796	0.8788	0.8780	0.8771	0.8763	0.8755	1	3	4	6	7
29	0.8746	0.8738	0.8729	0.8721	0.8712	0.8704	0.8695	0.8686	0.8678	0.8669	1	3	4	6	7
30	0.8660	0.8652	0.8643	0.8634	0.8625	0.8616	0.8607	0.8599	0.8590	0.8581	1	3	4	6	7
31	0.8572	0.8563	0.8554	0.8545	0.8536	0.8526	0.8517	0.8508	0.8499	0.8490	2	3	5	6	8
32	0.8480	0.8471	0.8462	0.8453	0.8443	0.8434	0.8425	0.8415	0.8406	0.8396	2	3	5	6	8
33	0.8387	0.8377	0.8368	0.8358	0.8348	0.8339	0.8329	0.8320	0.8310	0.8300	2	3	5	6	8
34	0.8290	0.8281	0.8271	0.8261	0.8251	0.8241	0.8231	0.8221	0.8211	0.8202	2	3	5	7	8
35	0.8192	0.8181	0.8171	0.8161	0.8151	0.8141	0.8131	0.8121	0.8111	0.8100	2	3	5	7	8
36	0.8090	0.8080	0.8070	0.8059	0.8049	0.8039	0.8028	0.8018	0.8007	0.7997	2	3	5	7	9
37	0.7986	0.7976	0.7965	0.7955	0.7944	0.7934	0.7923	0.7912	0.7902	0.7891	2	4	5	7	9
38	0.7880	0.7869	0.7859	0.7848	0.7837	0.7826	0.7815	0.7804	0.7793	0.7782	2	4	5	7	9
39	0.7771	0.7760	0.7749	0.7738	0.7727	0.7716	0.7705	0.7694	0.7683	0.7672	2	4	6	7	9
40	0.7660	0.7649	0.7638	0.7627	0.7615	0.7604	0.7593	0.7581	0.7570	0.7559	2	4	6	8	9
41	0.7547	0.7536	0.7524	0.7513	0.7501	0.7490	0.7478	0.7466	0.7455	0.7443	2	4	6	8	10
42	0.7431	0.7420	0.7408	0.7396	0.7385	0.7373	0.7361	0.7349	0.7337	0.7325	2	4	6	8	10
43	0.7314	0.7302	0.7290	0.7278	0.7266	0.7254	0.7242	0.7230	0.7218	0.7206	2	4	6	8	10
44	0.7193	0.7181	0.7169	0.7157	0.7145	0.7133	0.7120	0.7108	0.7096	0.7083	2	4	6	8	10
45	0.7071	0.7059	0.7046	0.7034	0.7022	0.7009	0.6997	0.6984	0.6972	0.6959	2	4	6	8	10
46	0.6947	0.6934	0.6921	0.6909	0.6896	0.6884	0.6871	0.6858	0.6845	0.6833	2	4	6	8	11
47	0.6820	0.6807	0.6794	0.6782	0.6769	0.6756	0.6743	0.6730	0.6717	0.6704	2	4	6	9	11
48	0.6691	0.6678	0.6665	0.6652	0.6639	0.6626	0.6613	0.6600	0.6587	0.6574	2	4	7	9	11
49	0.6561	0.6547	0.6534	0.6521	0.6508	0.6494	0.6481	0.6468	0.6455	0.6441	2	4	7	9	11
50	0.6428	0.6414	0.6401	0.6388	0.6374	0.6361	0.6347	0.6334	0.6320	0.6307	2	4	7	9	11
51	0.6293	0.6280	0.6266	0.6252	0.6239	0.6225	0.6211	0.6198	0.6184	0.6170	2	5	7	9	11
52	0.6157	0.6143	0.6129	0.6115	0.6101	0.6088	0.6074	0.6060	0.6046	0.6032	2	5	7	9	12
53	0.6018	0.6004	0.5990	0.5976	0.5962	0.5948	0.5934	0.5920	0.5906	0.5892	2	5	7	9	12
54	0.5878	0.5864	0.5850	0.5835	0.5821	0.5807	0.5793	0.5779	0.5764	0.5750	2	5	7	9	12
55	0.5736	0.5721	0.5707	0.5693	0.5678	0.5664	0.5650	0.5635	0.5621	0.5606	2	5	7	10	12
56	0.5592	0.5577	0.5563	0.5548	0.5534	0.5519	0.5505	0.5490	0.5476	0.5461	2	5	7	10	12
57	0.5446	0.5432	0.5417	0.5402	0.5388	0.5373	0.5358	0.5344	0.5329	0.5314	2	5	7	10	12
58	0.5299	0.5284	0.5270	0.5255	0.5240	0.5225	0.5210	0.5195	0.5180	0.5165	2	5	7	10	12
59	0.5150	0.5135	0.5120	0.5105	0.5090	0.5075	0.5060	0.5045	0.5030	0.5015	3	5	8	10	13

## Natural Cosines

Numbers in different columns to be subtracted, not added

°	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'	1'	2'	3'	4'	5'
	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°					
60	0.5000	0.4985	0.4970	0.4955	0.4939	0.4924	0.4909	0.4894	0.4879	0.4863	3	5	8	10	13
61	0.4848	0.4833	0.4818	0.4802	0.4787	0.4772	0.4756	0.4741	0.4726	0.4710	3	5	8	10	13
62	0.4695	0.4679	0.4664	0.4648	0.4633	0.4617	0.4602	0.4586	0.4571	0.4555	3	5	8	10	13
63	0.4540	0.4524	0.4509	0.4493	0.4478	0.4462	0.4446	0.4431	0.4415	0.4399	3	5	8	10	13
64	0.4384	0.4368	0.4352	0.4337	0.4321	0.4305	0.4289	0.4274	0.4258	0.4242	3	5	8	11	13
65	0.4226	0.4210	0.4195	0.4179	0.4163	0.4147	0.4131	0.4115	0.4099	0.4083	3	5	8	11	13
66	0.4067	0.4051	0.4035	0.4019	0.4003	0.3987	0.3971	0.3955	0.3939	0.3923	3	5	8	11	13
67	0.3907	0.3891	0.3875	0.3859	0.3843	0.3827	0.3811	0.3795	0.3778	0.3762	3	5	8	11	13
68	0.3746	0.3730	0.3714	0.3697	0.3681	0.3665	0.3649	0.3633	0.3616	0.3600	3	5	8	11	14
69	0.3584	0.3567	0.3551	0.3535	0.3518	0.3502	0.3486	0.3469	0.3453	0.3437	3	5	8	11	14
70	0.3420	0.3404	0.3387	0.3371	0.3355	0.3338	0.3322	0.3305	0.3289	0.3272	3	5	8	11	14
71	0.3256	0.3239	0.3223	0.3206	0.3190	0.3173	0.3156	0.3140	0.3123	0.3107	3	6	8	11	14
72	0.3090	0.3074	0.3057	0.3040	0.3024	0.3007	0.2990	0.2974	0.2957	0.2940	3	6	8	11	14
73	0.2924	0.2907	0.2890	0.2874	0.2857	0.2840	0.2823	0.2807	0.2790	0.2773	3	6	8	11	14
74	0.2756	0.2740	0.2723	0.2706	0.2689	0.2672	0.2656	0.2639	0.2622	0.2605	3	6	8	11	14
75	0.2588	0.2571	0.2554	0.2538	0.2521	0.2504	0.2487	0.2470	0.2453	0.2436	3	6	8	11	14
76	0.2419	0.2402	0.2385	0.2368	0.2351	0.2334	0.2317	0.2300	0.2284	0.2267	3	6	8	11	14
77	0.2250	0.2233	0.2215	0.2198	0.2181	0.2164	0.2147	0.2130	0.2113	0.2096	3	6	9	11	14
78	0.2079	0.2062	0.2045	0.2028	0.2011	0.1994	0.1977	0.1959	0.1942	0.1925	3	6	9	11	14
79	0.1908	0.1891	0.1874	0.1857	0.1840	0.1822	0.1805	0.1788	0.1771	0.1754	3	6	9	11	14
80	0.1736	0.1719	0.1702	0.1685	0.1668	0.1650	0.1633	0.1616	0.1599	0.1582	3	6	9	11	14
81	0.1564	0.1547	0.1530	0.1513	0.1495	0.1478	0.1461	0.1444	0.1426	0.1409	3	6	9	11	14
82	0.1392	0.1374	0.1357	0.1340	0.1323	0.1305	0.1288	0.1271	0.1253	0.1236	3	6	9	12	14
83	0.1219	0.1201	0.1184	0.1167	0.1149	0.1132	0.1115	0.1097	0.1080	0.1063	3	6	9	12	14
84	0.1045	0.1028	0.1011	0.0993	0.0976	0.0958	0.0941	0.0924	0.0906	0.0889	3	6	9	12	14
85	0.0872	0.0854	0.0837	0.0819	0.0802	0.0785	0.0767	0.0750	0.0732	0.0715	3	6	9	12	14
86	0.0698	0.0680	0.0663	0.0645	0.0628	0.0610	0.0593	0.0576	0.0558	0.0541	3	6	9	12	15
87	0.0523	0.0506	0.0488	0.0471	0.0454	0.0436	0.0419	0.0401	0.0384	0.0366	3	6	9	12	15
88	0.0349	0.0332	0.0314	0.0297	0.0279	0.0262	0.0244	0.0227	0.0209	0.0192	3	6	9	12	15
89	0.0175	0.0157	0.0140	0.0122	0.0105	0.0087	0.0070	0.0052	0.0035	0.0017	3	6	9	12	15
90	0.0000														



Quadrant	Angle	cos A =	Examples
First	0 to 90°	cos A	cos 33°26' = 0.8345
Second	90° to 180°	-cos(180° - A)	sin 146°34' = -cos(180° - 146°34') = -cos 33°26' = -0.8345
Third	180° to 270°	cos(A - 180°)	cos 213°26' = -cos(213°26' - 180°) = -cos 33°26' = -0.8345
Fourth	270° to 360°	cos(360° - A)	cos 326°34' = cos(360° - 326°34') = cos 33°26' = 0.8345

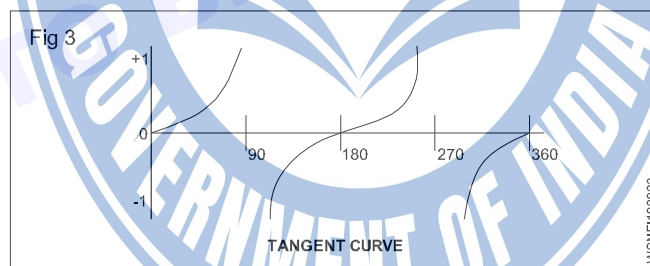
## Natural Tangents

°	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'	1'	2'	3'	4'	5'
	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°					
0	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157	3	6	9	12	15
1	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332	3	6	9	12	15
2	0.0349	0.0367	0.0384	0.0402	0.0419	0.0437	0.0454	0.0472	0.0489	0.0507	3	6	9	12	15
3	0.0524	0.0542	0.0559	0.0577	0.0594	0.0612	0.0629	0.0647	0.0664	0.0682	3	6	9	12	15
4	0.0699	0.0717	0.0734	0.0752	0.0769	0.0787	0.0805	0.0822	0.0840	0.0857	3	6	9	12	15
5	0.0875	0.0892	0.0910	0.0928	0.0945	0.0963	0.0981	0.0998	0.1016	0.1033	3	6	9	12	15
6	0.1051	0.1069	0.1086	0.1104	0.1122	0.1139	0.1157	0.1175	0.1192	0.1210	3	6	9	12	15
7	0.1228	0.1246	0.1263	0.1281	0.1299	0.1317	0.1334	0.1352	0.1370	0.1388	3	6	9	12	15
8	0.1405	0.1423	0.1441	0.1459	0.1477	0.1495	0.1512	0.1530	0.1548	0.1566	3	6	9	12	15
9	0.1584	0.1602	0.1620	0.1638	0.1655	0.1673	0.1691	0.1709	0.1727	0.1745	3	6	9	12	15
10	0.1763	0.1781	0.1799	0.1817	0.1835	0.1853	0.1871	0.1890	0.1908	0.1926	3	6	9	12	15
11	0.1944	0.1962	0.1980	0.1998	0.2016	0.2035	0.2053	0.2071	0.2089	0.2107	3	6	9	12	15
12	0.2126	0.2144	0.2162	0.2180	0.2199	0.2217	0.2235	0.2254	0.2272	0.2290	3	6	9	12	15
13	0.2309	0.2327	0.2345	0.2364	0.2382	0.2401	0.2419	0.2438	0.2456	0.2475	3	6	9	12	15
14	0.2493	0.2512	0.2530	0.2549	0.2568	0.2586	0.2605	0.2623	0.2642	0.2661	3	6	9	12	16
15	0.2679	0.2698	0.2717	0.2736	0.2754	0.2773	0.2792	0.2811	0.2830	0.2849	3	6	9	13	16
16	0.2867	0.2886	0.2905	0.2924	0.2943	0.2962	0.2981	0.3000	0.3019	0.3038	3	6	9	13	16
17	0.3057	0.3076	0.3096	0.3115	0.3134	0.3153	0.3172	0.3191	0.3211	0.3230	3	6	10	13	16
18	0.3249	0.3269	0.3288	0.3307	0.3327	0.3346	0.3365	0.3385	0.3404	0.3424	3	6	10	13	16
19	0.3443	0.3463	0.3482	0.3502	0.3522	0.3541	0.3561	0.3581	0.3600	0.3620	3	7	10	13	16
20	0.3640	0.3659	0.3679	0.3699	0.3719	0.3739	0.3759	0.3779	0.3799	0.3819	3	7	10	13	17
21	0.3839	0.3859	0.3879	0.3899	0.3919	0.3939	0.3959	0.3979	0.4000	0.4020	3	7	10	13	17
22	0.4040	0.4061	0.4081	0.4101	0.4122	0.4142	0.4163	0.4183	0.4204	0.4224	3	7	10	14	17
23	0.4245	0.4265	0.4286	0.4307	0.4327	0.4348	0.4369	0.4390	0.4411	0.4431	3	7	10	14	17
24	0.4452	0.4473	0.4494	0.4515	0.4536	0.4557	0.4578	0.4599	0.4621	0.4642	4	7	11	14	18
25	0.4663	0.4684	0.4706	0.4727	0.4748	0.4770	0.4791	0.4813	0.4834	0.4856	4	7	11	14	18
26	0.4877	0.4899	0.4921	0.4942	0.4964	0.4986	0.5008	0.5029	0.5051	0.5073	4	7	11	15	18
27	0.5095	0.5117	0.5139	0.5161	0.5184	0.5206	0.5228	0.5250	0.5272	0.5295	4	7	11	15	18
28	0.5317	0.5340	0.5362	0.5384	0.5407	0.5430	0.5452	0.5475	0.5498	0.5520	4	8	11	15	19
29	0.5543	0.5566	0.5589	0.5612	0.5635	0.5658	0.5681	0.5704	0.5727	0.5750	4	8	12	15	19
30	0.5774	0.5797	0.5820	0.5844	0.5867	0.5890	0.5914	0.5938	0.5961	0.5985	4	8	12	16	20
31	0.6009	0.6032	0.6056	0.6080	0.6104	0.6128	0.6152	0.6176	0.6200	0.6224	4	8	12	16	20
32	0.6249	0.6273	0.6297	0.6322	0.6346	0.6371	0.6395	0.6420	0.6445	0.6469	4	8	12	16	20
33	0.6494	0.6519	0.6544	0.6569	0.6594	0.6619	0.6644	0.6669	0.6694	0.6720	4	8	13	17	21
34	0.6745	0.6771	0.6796	0.6822	0.6847	0.6873	0.6899	0.6924	0.6950	0.6976	4	9	13	17	21
35	0.7002	0.7028	0.7054	0.7080	0.7107	0.7133	0.7159	0.7186	0.7212	0.7239	4	9	13	17	22
36	0.7265	0.7292	0.7319	0.7346	0.7373	0.7400	0.7427	0.7454	0.7481	0.7508	5	9	14	18	23
37	0.7536	0.7563	0.7590	0.7618	0.7646	0.7673	0.7701	0.7729	0.7757	0.7785	5	9	14	18	23
38	0.7813	0.7841	0.7869	0.7898	0.7926	0.7954	0.7983	0.8012	0.8040	0.8069	5	9	14	19	24
39	0.8098	0.8127	0.8156	0.8185	0.8214	0.8243	0.8273	0.8302	0.8332	0.8361	5	10	15	20	24
40	0.8391	0.8421	0.8451	0.8481	0.8511	0.8541	0.8571	0.8601	0.8632	0.8662	5	10	15	20	25
41	0.8693	0.8724	0.8754	0.8785	0.8816	0.8847	0.8878	0.8910	0.8941	0.8972	5	10	16	21	26
42	0.9004	0.9036	0.9067	0.9099	0.9131	0.9163	0.9195	0.9228	0.9260	0.9293	5	11	16	21	27
43	0.9325	0.9358	0.9391	0.9424	0.9457	0.9490	0.9523	0.9556	0.9590	0.9623	6	11	17	22	28
44	0.9657	0.9691	0.9725	0.9759	0.9793	0.9827	0.9861	0.9896	0.9930	0.9965	6	11	17	23	28
45	1.0000	1.0035	1.0070	1.0105	1.0141	1.0176	1.0212	1.0247	1.0283	1.0319	6	12	18	24	30
46	1.0355	1.0392	1.0428	1.0464	1.0501	1.0538	1.0575	1.0612	1.0649	1.0686	6	12	18	25	31
47	1.0724	1.0761	1.0799	1.0837	1.0875	1.0913	1.0951	1.0990	1.1028	1.1067	6	13	19	25	32
48	1.1106	1.1145	1.1184	1.1224	1.1263	1.1303	1.1343	1.1383	1.1423	1.1463	7	13	20	27	33
49	1.1504	1.1544	1.1585	1.1626	1.1667	1.1708	1.1750	1.1792	1.1833	1.1875	7	14	21	28	34
50	1.1918	1.1960	1.2002	1.2045	1.2088	1.2131	1.2174	1.2218	1.2261	1.2305	7	14	22	29	36
51	1.2349	1.2393	1.2437	1.2482	1.2527	1.2572	1.2617	1.2662	1.2708	1.2753	8	15	23	30	38
52	1.2799	1.2846	1.2892	1.2938	1.2985	1.3032	1.3079	1.3127	1.3175	1.3223	8	16	24	31	39
53	1.3270	1.3319	1.3367	1.3416	1.3465	1.3514	1.3564	1.3613	1.3663	1.3713	8	16	25	33	41
54	1.3764	1.3814	1.3865	1.3916	1.3968	1.4019	1.4071	1.4124	1.4176	1.4229	9	17	26	34	43
55	1.4281	1.4335	1.4388	1.4442	1.4496	1.4550	1.4605	1.4659	1.4715	1.4770	9	18	27	36	45
56	1.4826	1.4882	1.4938	1.4994	1.5051	1.5108	1.5166	1.5224	1.5282	1.5340	10	19	29	38	48
57	1.5399	1.5458	1.5517	1.5577	1.5637	1.5697	1.5757	1.5818	1.5880	1.5941	10	20	30	40	50
58	1.6003	1.6066	1.6128	1.6191	1.6255	1.6319	1.6383	1.6447	1.6512	1.6577	11	21	32	43	53
59	1.6643	1.6709	1.6775	1.6842	1.6909	1.6977	1.7045	1.7113	1.7182	1.7251	11	23	34	45	56

## Natural Tangents

°	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'	1'	2'	3'	4'	5'
	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°					
60	1.7321	1.7391	1.7461	1.7532	1.7603	1.7675	1.7747	1.7820	1.7893	1.7966	12	24	36	48	60
61	1.8040	1.8115	1.8190	1.8265	1.8341	1.8418	1.8495	1.8572	1.8650	1.8728	13	26	38	51	64
62	1.8807	1.8887	1.8967	1.9047	1.9128	1.9210	1.9292	1.9375	1.9458	1.9542	14	27	41	55	68
63	1.9626	1.9711	1.9797	1.9883	1.9970	2.0057	2.0145	2.0233	2.0323	2.0413	15	29	44	58	73
64	2.0503	2.0594	2.0686	2.0778	2.0872	2.0965	2.1060	2.1155	2.1251	2.1348	16	31	47	63	78
65	2.1445	2.1543	2.1642	2.1742	2.1842	2.1943	2.2045	2.2148	2.2251	2.2355	17	34	51	68	85
66	2.2460	2.2566	2.2673	2.2781	2.2889	2.2998	2.3109	2.3220	2.3332	2.3445	18	37	55	73	92
67	2.3559	2.3673	2.3789	2.3906	2.4023	2.4142	2.4262	2.4383	2.4504	2.4627	20	40	60	79	99
68	2.4751	2.4876	2.5002	2.5129	2.5257	2.5386	2.5517	2.5649	2.5782	2.5916	22	43	65	87	108
69	2.6051	2.6187	2.6325	2.6464	2.6605	2.6746	2.6889	2.7034	2.7179	2.7326	24	47	71	95	119
70	2.7475	2.7625	2.7776	2.7929	2.8083	2.8239	2.8397	2.8556	2.8716	2.8878	26	52	78	104	131
71	2.9042	2.9208	2.9375	2.9544	2.9714	2.9887	3.0061	3.0237	3.0415	3.0595	29	58	87	116	145
72	3.0777	3.0961	3.1146	3.1334	3.1524	3.1716	3.1910	3.2106	3.2305	3.2506	32	64	96	129	161
73	3.2709	3.2914	3.3122	3.3332	3.3544	3.3759	3.3977	3.4197	3.4420	3.4646	36	72	108	144	180
74	3.4874	3.5105	3.5339	3.5576	3.5816	3.6059	3.6305	3.6554	3.6806	3.7062	41	81	122	163	204
75	3.7321	3.7583	3.7848	3.8118	3.8391	3.8667	3.8947	3.9232	3.9520	3.9812	46	93	139	186	232
76	4.0108	4.0408	4.0713	4.1022	4.1335	4.1653	4.1976	4.2303	4.2635	4.2972	53	107	160	213	267
77	4.3315	4.3662	4.4015	4.4374	4.4737	4.5104	4.5483	4.5864	4.6252	4.6646					
78	4.7046	4.7453	4.7867	4.8288	4.8716	4.9152	4.9594	5.0045	5.0504	5.0970					
79	5.1446	5.1929	5.2422	5.2924	5.3435	5.3955	5.4486	5.5026	5.5578	5.6140					
80	5.6713	5.7297	5.7894	5.8502	5.9124	5.9758	6.0405	6.1066	6.1742	6.2432					
81	6.3138	6.3859	6.4596	6.5350	6.6122	6.6912	6.7720	6.8548	6.9395	7.0264					
82	7.1154	7.2066	7.3002	7.3962	7.4947	7.5958	7.6996	7.8062	7.9158	8.0285					
83	8.1443	8.2636	8.3863	8.5126	8.6427	8.7769	8.9152	9.0579	9.2052	9.3572					
84	9.5144	9.677	9.845	10.02	10.20	10.39	10.58	10.78	10.99	11.20					
85	11.43	11.66	11.91	12.16	12.43	12.71	13.00	13.30	13.62	13.95					
86	14.30	14.67	15.06	15.46	15.89	16.35	16.83	17.34	17.89	18.46					
87	19.08	19.74	20.45	21.20	22.02	22.90	23.86	24.90	26.03	27.27					
88	28.64	30.14	31.82	33.69	35.80	38.19	40.92	44.07	47.74	52.08					
89	57.29	63.66	71.62	81.85	95.49	114.6	143.2	191.0	286.5	573.0					
90	x														

DIFFERENCES  
UNTRUSTWORTHY  
HERE



Quadrant	Angle	$\tan A =$	Examples
First	0 to 90°	$\tan A$	$\tan 56^\circ 17' = 1.4986$
Second	90° to 180°	$-\tan(180^\circ - A)$	$\tan 123^\circ 43' = -\tan(180^\circ - 123^\circ 43')$ $= -\tan 56^\circ 17' = -1.4986$
Third	180° to 270°	$\tan(A - 180^\circ)$	$\tan 236^\circ 17' = \tan(236^\circ 17' - 180^\circ)$ $= \tan 56^\circ 17' = 1.4986$
Fourth	270° to 360°	$-\tan(360^\circ - A)$	$\tan 303^\circ 43' = -\tan(360^\circ - 303^\circ 43')$ $= -\tan 56^\circ 17' = -1.4986$